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Hamstring Assistance Device

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By

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Abstract

Reduced functionality of the hamstring muscle creates a need for a device to assist in riding a bicycle. The project is to design a device to enable a person who has reduced usage of the hamstring muscle to ride a bicycle by aiding leg motion between the pedal angles of 180° to 225°. The device is for recreational cycling, and adaptable to a range of bicycles. Design concepts were made to store energy provided during the portion of the pedal stroke in which the user can provide power (0°-180°), so it could be utilized during the portion of the pedal stroke where the user cannot provide power due to their disability (180°-225°). The final design uses a pair of extension springs with a four stage pulley system to store energy and provide the range of motion for the pedal stroke.

Table of Contents

List of Figures

List of Equations

Introduction

Bicycling is not only a great form of alternative transportation, but also a wonderful recreational activity. Unfortunately many people experience injury which may render them unable to ride a bicycle, one of the more common injuries among Americans is that of a stroke. After stroke there can be lasting effects on its victim such as hemiparesis which is a partial paralysis of one side of the body. It has been shown that bicycle riding can be used as an effective form of physical therapy for such victims. Hemiparesis victims while, may be capable of riding a stationary bicycle, may not have the necessary muscular control to complete a full bicycle crank cycle on a recreational bicycle, in particular these victims whose hemiparesis has affected their hamstring muscles will have increased difficulty in completing a portion of the crank cycle in which the hamstring muscle is the only active muscle. The goal of this project is to design and build a device to enable a user who has reduced usage of the hamstring to ride a bicycle by aiding leg motion between the crank angles of 180° to 225° (the angles of a crank cycle in which the hamstring is the only active muscle). This device must be useable for recreational purposes, and must be adaptable to a range of bicycles. This report will explain the methods used to create the device as well as provide information on the end product, conclusions and suggestions for future work on this problem.

Background

“Stroke is the leading cause of serious, long-term disability in the United States” (Weiss, T. C). One of these long term disabilities is known as hemiparesis, which is a partial paralysis of one side of the body. Approximately 8 out of 10 stroke survivors suffer from some form of hemiparesis (Hemiparesis, 2014). In addition to stroke, many diseases of the nervous system or brain, such as multiple sclerosis or brain tumors, can cause hemiparesis (Weiss, T. C.). With hemiparesis, the disabled person still has the ability to operate the affected side of their body; however, this operation suffers from reduced muscular strength (Hemiparesis). There are many accepted treatment options for those who suffer from hemiparesis, some involve electrical stimulation, while others focus more on physical rehabilitation of the affected muscle groups (Hemiparesis).

Whichever treatment method, or combination thereof, is chosen for rehabilitation after a stroke, it is important to understand the concepts of both neuroplasticity and neurogenesis. Neuroplasticity is the brain’s ability to “rewire” itself after being damaged. Neurogenesis refers to the brain’s ability to create new neurons. These neurons will require support from neighboring cells, blood supply, and connection with other neurons for survival. (Neuroplasticity and Stroke Recovery). “Rehabilitation involving neuroplasticity principles requires repetition of task and task specific practice to be effective” (Neuroplasticity and Stroke Recovery). What this means for stroke patients is that the more the patient is able to use, or practice stimulating, the affected areas of the body and mind, the more effective rehabilitation will be. (Neuroplasticity and Stroke Recovery).

For patients suffering from lower extremity weakness, the pedaling motion required to ride a bicycle could be an effective repetitive motion to strengthen all muscles in the leg. Utilizing the concept of neuroplasticity and its application to stroke patients, it would be desirable for a patient to have the ability to incorporate bicycling into everyday life, perhaps even as a primary mode of transportation. Due to excessive weakness of any or all muscles in the leg, it may be impossible to use a bicycle without some form of pedal assistance.

Studies have shown that muscles work in a systematic and coordinated way during cycling to direct power from the human body to the pedal and crank of the bicycle. This is known as muscle recruitment and there have been exhaustive studies researching the patterns of how humans use their muscles while cycling. We have studied this literature and identified critical areas where force production by a rider with hemiparesis, specifically affecting their hamstring muscles, would be weakened or impossible.

The crank cycle can be broken down into three phases, the propulsive or downstroke phase, the recovery or upstroke phase and the pushing phase, in which the foot is pushed forward at the top dead center (TDC).

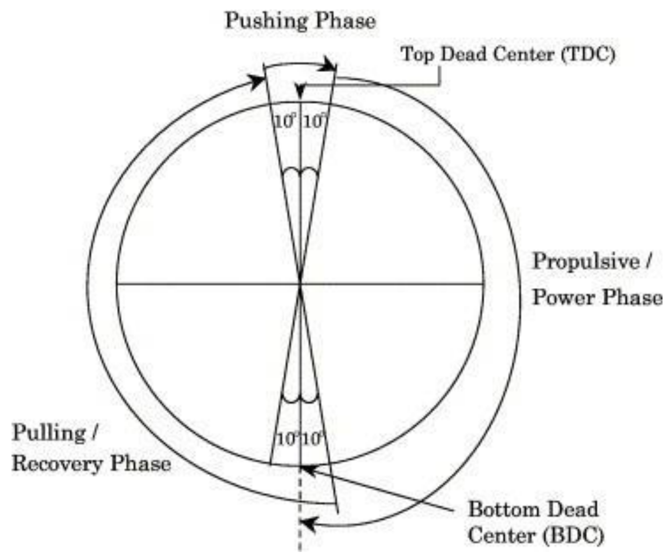

Figure 1: Crank Cycle Phases

The propulsive phase, beginning at the top of the stoke (0°), is characterized by the extension of the hip and extension of the knee. These motions primarily recruit the quadriceps and glutes. During the recovery phase, from 180° to 360°, the hamstrings and hip flexors work together to flex both the hip and knee. Force generated by these muscles serve two functions. The first is to reduce the resistance on the crank for the propulsive phase on the opposite leg. The second, the recovery phase provides flexion allowing the crank to rotate and assist the opposite leg in propulsion.

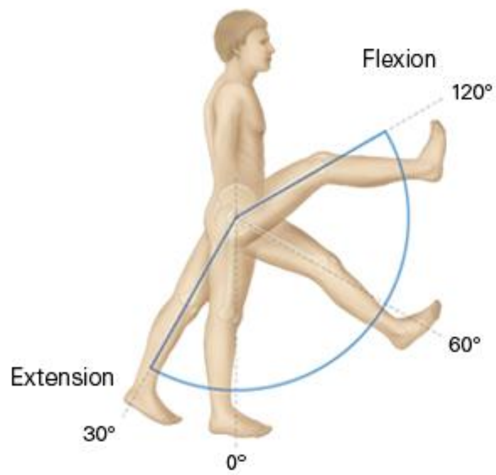

Figure 2: Hamstring Leg Motion

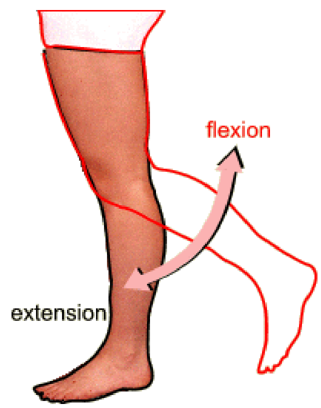

Figure 3: Hamstring Calf Motion

The primary focus of our project, the hamstring muscle group, is comprised of three different muscles, The Bicep femoris, the semimembranosus and the semitendinosus. This group is active from 10°-350° at varying intensities. The Bicep femoris is active from 350°-230°, with peak output at 110°. The semimembranosus and semitendinosus are active from 10°-230°, with a peak output at 100°. During the last 45° of hip extension range of motion, ending just past the 180° point of the crank cycle, at the beginning of the recovery phase, the hamstrings work alone in hip extension. This is where riders with weak hamstrings are unable to continue applying torque to the pedal in the recovery phase and must attempt to complete the crank cycle using torque generated by the other leg, which is in the propulsion stage.

With a weak hamstring, it may still be possible for a rider to cause a full revolution of the crank by using down-stroke force from the opposite leg. However there is an importance to a pedal stroke for where there is even distribution of power to the pedals during the course of the entire pedal revolution. The use of racing pedals that restrain the foot to the pedal are an example of this concept. These restraints help the rider to exert force throughout the full revolution by allowing them to fully engage their active muscles, including the hamstrings, on the upstroke of the pedal cycle. Muscles activated in a defined pattern are not just for optimizing the energy transfer from the human body to the machine but also provide protection to the major joints. For this reason we look to design a device to aid the hamstring in generating force on the pedal during the recovery phase of the pedal stroke, to help riders with hemiparesis have an improved cycling experience and even reduce the risk of injury.

So far, many devices have been designed to aid rehabilitation of weakened muscles due to hemiparesis, but none simply for recreational purposes. Many are based on the premise of actuating the weaker muscles against incrementally increasing resistance. One such device is US patent 6220991B1, depicted in Figure 4.

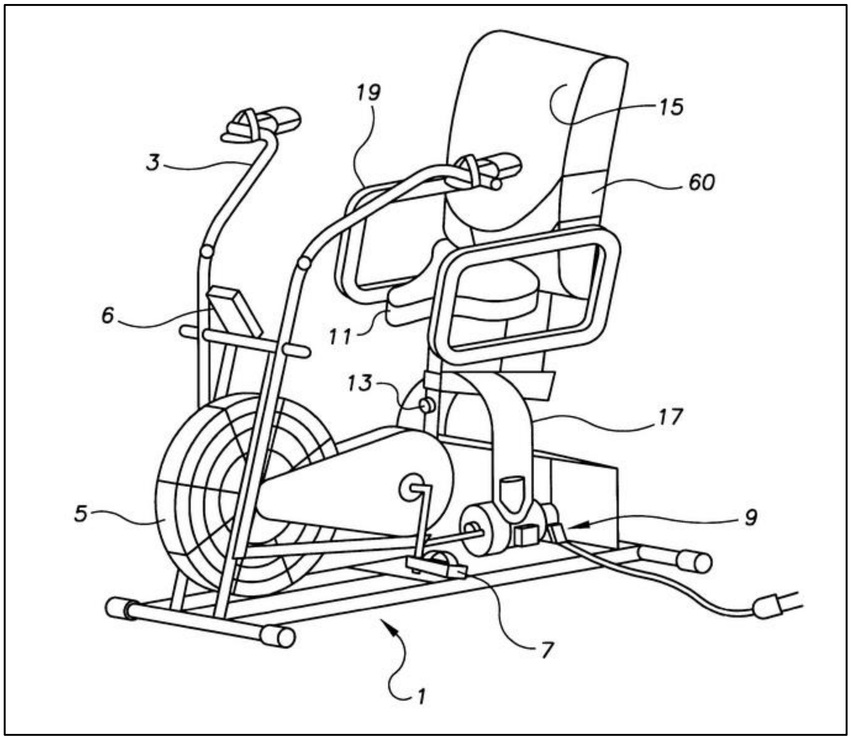

Figure 4: Depiction of Device From US patent 6220991B1

The user grips the handles and places his or her feet in the pedal straps. They would then rotate the pedals. The pedals are connected to the handles, causing some motion. They are also connected to a fan blade in the front of the device, which provides slight resistance. This device allows for safe exercise for a person with reduced muscle strength, such as in the case with hemiparesis. Conventional exercises such as jogging or weight lifting can be too strenuous for a person with reduced muscle strength. This device has the advantage of allowing for varying loads as well as providing a controlled experience.

With respect to the proposed goal, devices exist that could be used to enhance the use of a bicycle for recreational or competitive activity. One such device is US patent 5515746A, depicted in Figure 5.

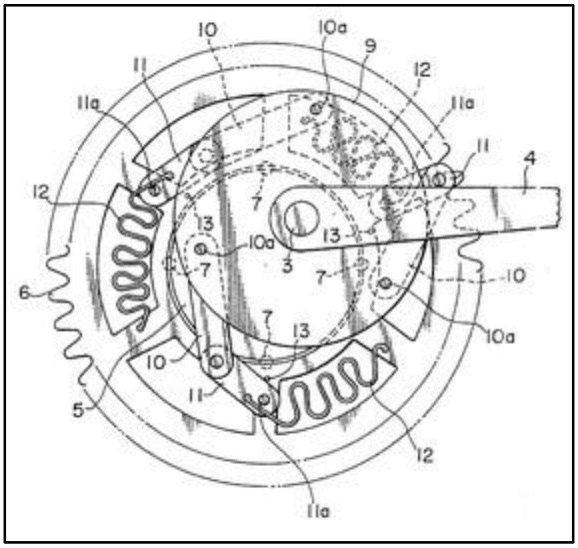

Figure 5: Depiction of Device From US patent 5515746A

This device was designed to increase torque to the drive shaft without increasing the length of the crank arm. It consists of a "bearing disposed on a frame of the bicycle and has an eccentric plate put on a rotating shaft having cranks with pedal" (U.S. Patent No. 5515746A, 1994). The plate is fixed relative to the frame of the bicycle. There are linking members connecting the chain wheel to a working plate. The result is that the chain wheel can be rotated in a smaller radius, resulting in greater torque. This device could be useful for a person with hemiparesis because the person would produce reduced torque for sections of the cycle.

Another type device to allow the use of a bicycle for a person with hemiparesis would be one that attached to the user. This is seen is U.S. Patent No. 6368256, depicted in Figure 6.

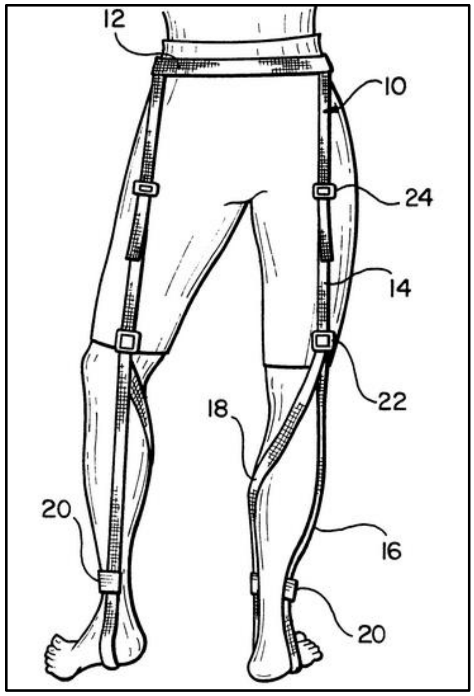

Figure 6: Depiction of Device From U.S. Patent No. 6368256

As can be seen, there are elastic bands that reach from the waist to the feet. During the downward motion of the pedaling action, the bands are stretched, storing energy. This energy is released during the upward motion, assisting the user. For our particular case of hemiparesis, the subject has a weakened hamstring. This results in difficulty raising up the leg in the backward motion of the stroke. This device would take energy from the stronger sections of the legs and use it for the problematic section.

Before beginning discussion on the design of a device for a bicycle, let us first define some basic bicycle terminology, for this please refer to Figure 7 below:

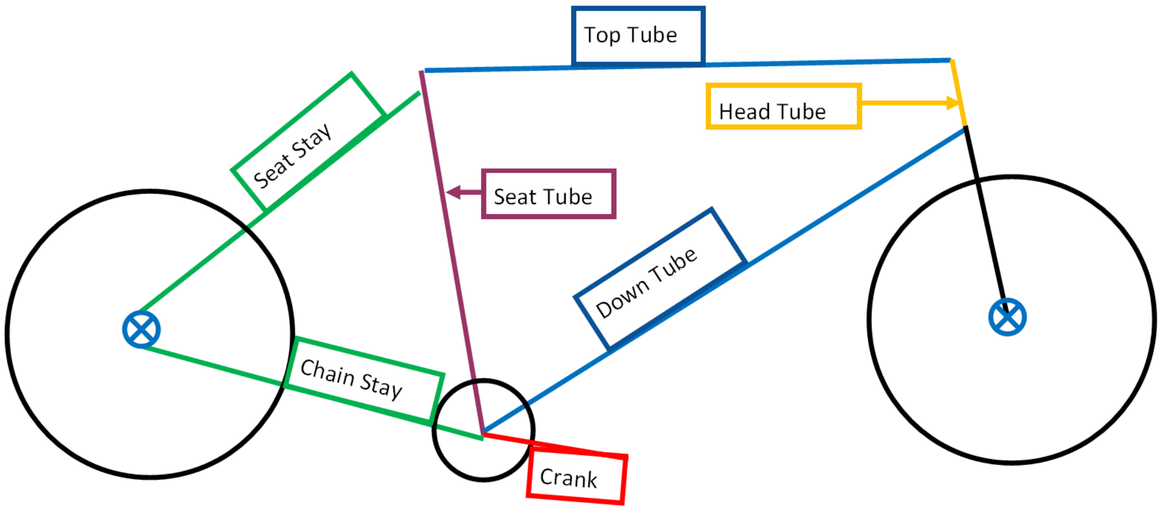

Figure 7: Bike Frame Terminology

In the following discussions we refer to a section of the bike as “the rear triangle” this is comprised of the seat tube, the chain stay, and the seat stay.

Functional Requirements

This section lists the functional requirements developed by the project team. These requirements act as the design specifications for the final device. Each requirement is followed by a short explanation of its significance to the problem we have outlined in previous sections. They are used in the design matrix to rank each device on how well it meets each functional requirement.

- Must be adaptable to a range of commuter/road bikes bicycles.
  - The team wants the device to be useful to people who already have a bicycle so they don't have to pay for an expensive new one.
- Must interface with bike and/or rider(allow for free motion of the bike)
  - The goal is to design an assistive device for recreational purposes. This specification is to ensure that our design will allow a bicycle go anywhere a non assisted bicycle would normally go, rather than be stationary or relient on a source of power.
- Must aid leg motion between the angles of 180°(bottom of pedal cycle) to 225°.
  - The users that are the focus of the project cannot generate enough force with their hamstring to pull their leg through the phase denoted by the angles. These numbers were found through research that stated 180° to 225° is the range at which the hamstring is the primary muscle engaged
- Powered solely by the rider of the bike
  - Team team does not want did not want to use any electrical components to make the device. These require expensive components, increase weight and would cause the device to rely on a power source.
- Must cost under $100 to produce.
  - Each student is allotted $150 though the MQP program at Worcester Polytechnic Institute. This device budget was decided to allow for the creation of multiple prototypes

should one design fail to work in practice, or should a prototype break during testing. This will also potentially allow for a series of design iterations on a successful design as well.

- Must weigh under 5lb
  - The device cannot increase the weight of the bike and rider system by a large margin or it will increase the force required on the pedal to continue forward motion, negatively affecting our goal. However the device must have some weight so it was found through research that the average weight of a commuter bike is 30lb and it was decided that 15% increase in weight would be acceptable, which left us with 5lb.
- Must retain the weight balance that is characteristic of the bicycle.
  - The use of a bicycle relies on it being a balanced devices that does not tip. The device must not cause major change in the bicycles balance as we don't want a user that has difficulty riding a bicycle to have to operate an imbalance bicycle.
- Must be smaller than 50cm by 38cm by 12cm
  - Based on the geometry of an average commuter bicycle the team decided that this would be the most appropriate design envelope without inhibiting the normal operation of the bicycle as a device of this size would fit within the frame of the bike.
- Must not interfere with pedaling motion
  - One of the challenges of this problem is working around the mechanisms that make up a bicycle. Our device must avoid that path of any preexisting components that are in motion while riding including the wheels, crank and chain. The device must also allow the leg to be engaged with the pedal at all times as well as avoid the leg entirely for as it has potential to cause injury.
- Easily removable with basic tools

    - This allows a user to attach the device to a bicycle without having to pay for unconventional tools. Some tools that would be considered basic are a screwdriver, wrench, pliers, hammer or allen wrench.
- Must comply with (ASME) ASTM standard G50-10 for weather resistant material
    - The device to be designed is for a recreational bicycle rather than a stationary bicycle so it should be able to withstand the weathering conditions bicycles are typically subjected to.
- Must generate positive tangential force sufficient to supply motion through the range of 180° to 225°
    - Tangential force, relative to the path of the crank, on the pedal is needed to accelerate and maintain speed of the bicycle. The force on the pedal was found through experimentation using a spring scale on the bicycle pedal as well as mathematical evaluation of the bike-rider system with a desired acceleration of the rider.
- Producible in WPI laboratories
    - The team only has access to the equipment in WPI laboratories so we would be limited to the manufacturing tools available to us. This must be considered when developing a design to ensure that it will be producible.
- Means of controlling and inducing proper ankle position while keeping the foot attached to the pedal
    - One of the issues hemiparesis patients suffer from, is an inability to maintain ankle alignment. The alignment of the ankle relative to the pedal affects the amount of power one is able to transmit to the pedal.

Preliminary Design

Two Chains Design Concept

One design that was conceptualized was assigned the name of "Two Chains" design, this concept was based off of a regenerative braking system for bicycles in which the bicycle momentum would be stored in a flywheel, when braking, and released back to the wheel when needed for acceleration. The two chains design, was essentially this regenerative braking system without a flywheel for energy storage, rather the design would transmit the rotational energy through a gearing system directly to the bicycle cranks. Depicted below in Figure 8 is a representation of this design.

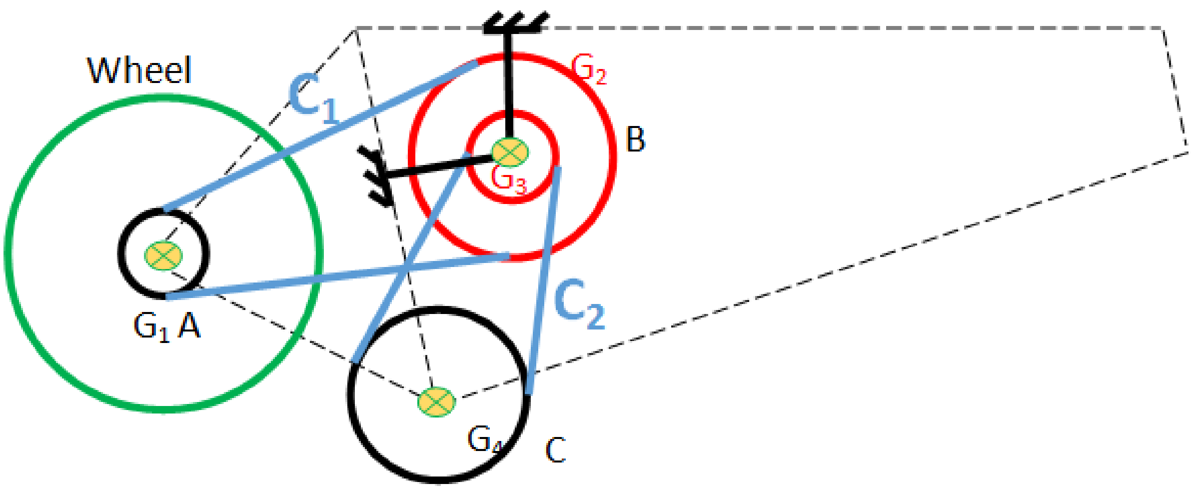

Figure 8: Drawing of "Two Chains" Design Concept

Seen above in Figure 8 is the design concept for the two chains design. In this design rotational energy from the wheel is transmitted from gear 1 (G1 in figure) through the first chain (labeled C1 above), to gear 2 (G2 above). The gearing here was designed to take the high angular velocity of the wheel as an input, and gear it down to a low angular velocity, high torque output. Gear 2 was then directly attached to gear 3 (G3 above), thus allowing gear three to rotate at the same angular velocity as gear 2. Gear 3 would than transmit its rotational energy through chain 2 (C2 above) to gear 4 (G4 above). The gearing in this section of the design was meant to lower the angular velocity of gears 2 and 3, and generate a higher torque output at gear 4, thus ensuring the device would be able to lift the user's leg through the desired crank angle, without causing the crank to rotate too quickly. During the conceptualization of this design, it was discussed that a coasting mode would be desirable, should the user wish to stop pedaling, while remaining in motion. It was also discussed that we should consider implementing a gear cassette allowing for variable gear ratios, and subsequently variable output angular velocities and output torques to the crank.

Ultimately it was decided that it would cost the most to produce, would be less adaptable to a range of bikes, and would be difficult for a laymen to install on their personal bikes, for these reasons the design was rejected. These reasons were tabulated in a decision matrix to compare this concept to others. The decision matrix can be found in Appendix H: Decision Matrix.

Four Bar Design Concept

Another design that was considered was a quick return linkage mechanism. In this design, a bar would be mounted directly to the pedal shaft and connect to a spring through the means of a four bar mechanism. Energy would be stored in the spring during the downward phase of the pedal stroke where the rider is able to apply force, this energy would then be released during the upward phase of the pedal stroke which the rider has difficulty completing due to their disability. The four bar would be designed so that the section of the stroke that stores energy would be a greater angle than the section of the stroke that the energy is released in. This way, the returned force would be concentrated in the portion of the stroke needed. This would be designed with the same method as a quick return mechanism, a simple representation of this device can be seen below in Figure 9

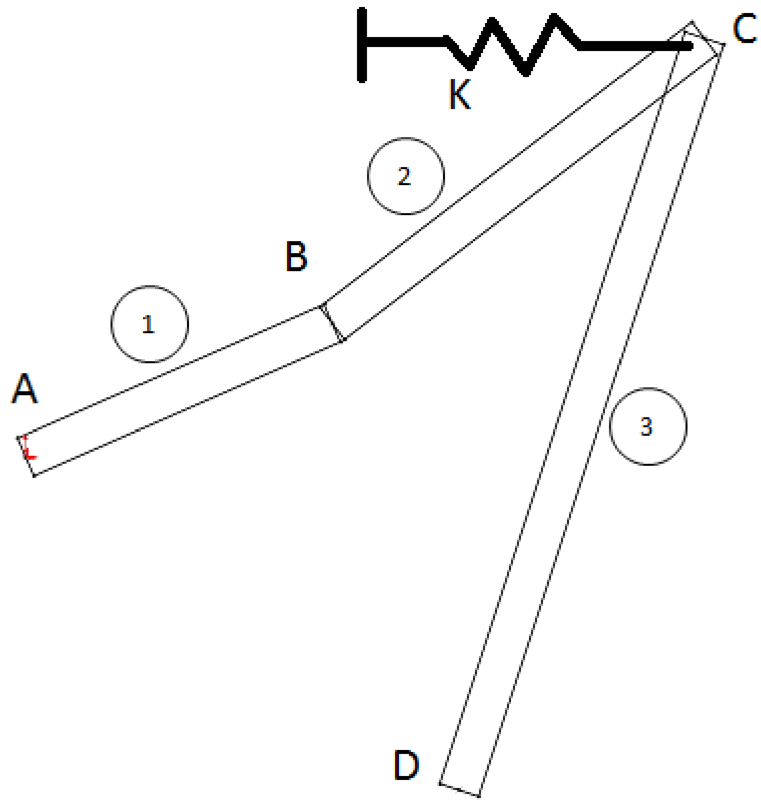

Figure 9: Depiction of "Four Bar" Design Concept

In the figure above, the mechanism has pin joints at points A, B, C and D. Link 1 represents the crank of the bicycle, while links 2 and 3, make up the remainder of the quick return mechanism. The addition of the spring in this mechanism, allows for an appropriate amount of energy to be stored during the pushing phase of the pedal stroke, which would be released during the portion of pedal stroke the rider needed assistance with.

This design scored poorly relative to the other designs in our decision matrix because it poorly met many of the design requirements. For example, it had a low score in "adaptable to a range of bikes”. This is because the angles were dependent on the size of the bike and attaching the mechanism is dependent on the shape of the bicycle. In addition, the bulky nature of the device caused it to score poorly in “Doesn't interfere with motion of bike” and “Maintain weight balance of bike.” The design would require long beams that would need to reciprocate rapidly. Such movement would be isolated to one side of the bicycle, causing imbalance. This also caused concerns for the weight of the device, lowering its score in the weight requirement.

The device did score well in some other areas, particularly ones related to the functionality of the device. It did increase the power to the pedal and it was powered solely by the rider. It could be made with materials that would meet weather resistance standards. Like the other devices, it scored well in force applied, attach foot to pedal, and force application. This suggests that certain aspects of this design could be integrated into future designs.

Retractor Design Concept

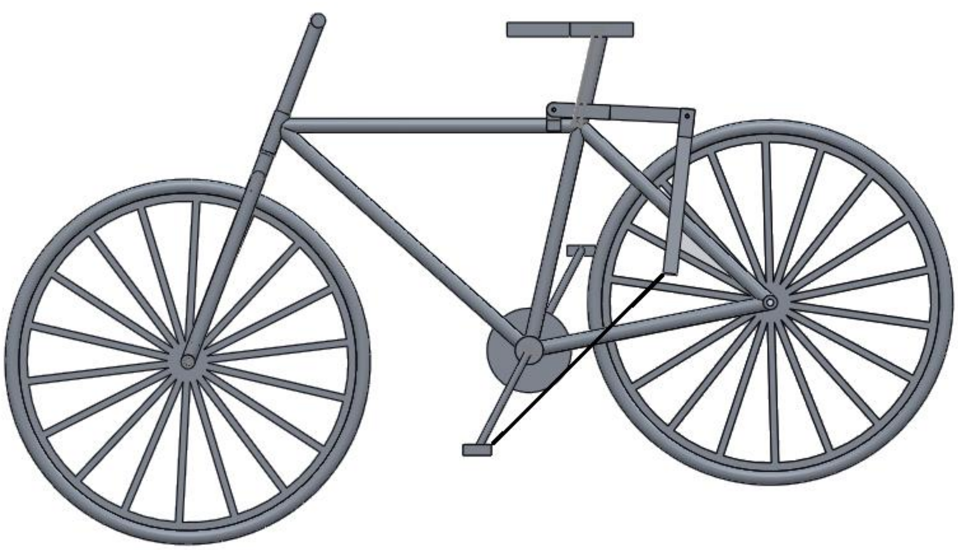

Figure 10: Depiction of "Retractor" Design Concept

The retractor design concept was based on the motion of a tape measure as it pulls in the tape. A design concept was created that would use a spring placed behind the seat of the bike used to pull on the pedal of the bike to cause normal motion of the pedal through the phase of the stroke where the leg is weak. A steel cable would be connected to the spring and the pedal. During the downstroke the wire would be pulled causing deflection in the spring. The stored energy would then be released as the pedal continues in the cycle and the spring returns to a neutral position. This design out scored the other designs significantly in the decision matrix with a score of 979 (see Appendix H: Decision Matrix) due to many attractive features. Its mounting placement makes it highly adaptable to different bikes and gave the designer plenty of room to design the features of the device without interfering with the normal motion of the bike and rider. Also due to the small number of parts it seemed like the most inexpensive and simple way to achieve the desired force outputs on the bicycle crank.

A variety of spring options were explored including a radial torsion spring, an extension spring and finally a torsion bar.

Figure 11: Radial Torsion Spring Example

Figure 12: Extension Spring Example

Figure 13: Torsion Bar Example

Design Analysis & Refinement

After measuring that the distance between the pedals at two opposite points along the cycle was 16 inches, we concluded that the spring must allow for 16 inches of displacement. This design requirement immediately eliminated the spiral spring from the list of options as there are no available spiral springs with that amount of displacement that were strong enough for our application. Manufacturing a custom spring was briefly considered but given that there were other spring options concerns about the costs and challenges of designing such a part deterred us from exploring the option further.

Our team concluded that the torsion bar would likely be the best option for our design as it took up very little space and would be very inexpensive. Also with the simple addition of a lever arm at the non-fixed end of the torsion bar we would be able to easily adjust the linear motion of the wire based on the angle of deflection that was attainable from the torsion bar. However designing the torsion bar proved to be far more challenging than expected.

The first task in designing the torsion bar, was to determine the forces the torsion bar would need to exert on the system to assist the rider through the full motion of the pedal stroke. We began by determining an appropriate acceleration for the bicycle to undergo due to the assisting force of the torsion bar, this value was obtained from table 7-1 of Robert L. Norton's Design of Machinery text, the team used a value of 0.1g, a common acceleration encountered during gentle acceleration in an automobile. Combining this acceleration with the mass of the system (rider mass + bike mass + device mass) we could calculate the necessary force to be applied to the bike, using Newton's second law, this force was found to be 18.5lbs. After taking into account the rolling friction of the wheels, and assuming the rider was in a top gear, we determined the necessary force to be acting on the crank would be approximately 27lbs, this amount of force would cause the bike to accelerate at our desired 0.1g's (detailed calculations can be found in Appendix B: Force Required to Accelerate Bike).

After determining the force required to pull on the crank to achieve our desired acceleration, we conducted a four-bar analysis to determine the amount of torque the torsion bar would need to generate in-order to pull the crank with the desired force through the crank phase in question, 180-225 degrees. We determined the max torque the torsion bar needed to be capable of generating, using an 18 inch lever arm, would be 610 inch-pounds (a detailed four-bar mechanism analysis and torsion bar calculations can be found in Appendix C: Four-Bar Analysis and Torsion Bar Calculations).

The three most important factors we had to consider for the design of the torsion bar where, the angle of twist produced by the torque on the torsion bar, the stresses at the mounting ends of the torsion bar, and the safety factor of the torsion bar. To calculate the angle of twist we used Equation 1 below:

$$\theta = \frac{Tl}{JG}$$

Equation 1: Torsion Bar Angle of Twist

In Equation 1, theta represents the angle of twist, or angular deflection experienced by the torsion bar, T the torque applied to the torsion bar, *l* the length of the torsion bar, J the polar area moment of inertia, and G the modulus of rigidity (shear modulus). The modulus of rigidity is a material property and was changed according to which material we were considering. To calculate the stress of the torsion bar, we used Equation 2, below:

$$S_t = K_{fs}\left(\frac{16T}{\pi D^3}\right)$$

Equation 2: Stress in a Torsion Bar

Where K_{fs} is the fatigue stress concentration factor, T is the torque the torsion bar is subjected to, and D is the equivalent diameter of the square mounting end on the torsion bar. The equivalent diameter is calculated through equations 6.7 c, and 6.7 d, from Norton's Machine Design Text reproduced below as Equation 3 and Equation 4.

$$A_{95} = \pi\left[\frac{d^2-(0.95d)^2}{4}\right]$$

Equation 3: Area of Non-Round Cross Section for Equivalent Diameter

$$D_{equiv} = \sqrt{\frac{A_{95}}{0.0766}}$$

Equation 4: Equivalent Diameter for Non-Round Cross Section

Where A_{95} represents the portion of the cross-sectional area of the non-round part that is stressed between 95% and 100% of its maximum stress. And D_{equiv} is the equivalent diameter of a non-circular rotating cross section, the square mounting ends of the torsion bar for us. The torsion bar we were designing had a square mounting end, which was smaller than the diameter of the torsion bar itself; this change in geometry causes a disruption in the flow of force through the part, and as such will concentrate forces at the interface of the two geometries. K_{fs}, the fatigue stress concentration factor, accounts for the increased stress due to the change in geometry when the part is subjected to dynamic loading, and can be calculated with equation 6.11b from Norton's Machine Design text reproduced below, as Equation 5:

$$K_{fs} = 1 + q(K_t - 1)$$

Equation 5: Fatigue Stress Concentration Factor

Where q is the notch sensitivity and K_t is the geometric stress concentration factor. The notch sensitivity is defined in equation 6.13 of Norton's Machine Design text, reproduced below as Equation 6:

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$

Equation 6: Notch Sensitivity

Where a represents Neuber's constant, and r the notch radius. The notch radius, refers to the radius of curvature of the edge between two sections of differing geometries. Neuber's constant can be determined from tables 6-6, 6-7, and 6-8 of Robert L. Norton's Machine Design text book, for the materials we were using we needed to interpolate these tables, so we could get Neuber's constants for tensile strengths not listed in the tables (interpolation of these tables can be found in Appendix F: Interpolations for Determining Stress Concentration Factors). The geometric stress concentration factor used in Equation 5 can be calculated using:

$$K_t = A\left(\frac{r}{d}\right)^b$$

Equation 7: Geometric Stress Concentration Factor

Where d is the diameter of the smaller geometry, and r is the radius of curvature between the two geometries; A and b represent constants which can be determined from tables in appendix C of Robert L. Nortons Machine Design text book, once again to get the values for our specific geometry we had to interpolate these tables, this interpolation can be found in Appendix F: Interpolations for Determining Stress Concentration Factors.

Lastly the safety factor of the torsion bar can be determined by the ratio between the yield strength of the material and the stresses in the material, for our purposes we used Equation 8 below:

$$n = \frac{\pi D^3}{32K_{fs}\left(\frac{T_a}{S_n} + \frac{T_m}{S_y}\right)}$$

Equation 8: Safety Factor for a Torsion Bar

Where D is the diameter of the torsion bar, K_{fs} is the fatigue stress concentration factor, T_a the alternating torque, T_m the mean torque, S_n the fatigue limit of the material, and S_y the yield strength of the material. Since our torsion bar operates between zero torque and max torque, only being rotated in one direction, it is subjected to repeated loading, for which the alternating and mean torques are equivalent. The fatigue limit of the torsion bar is determined by taking into account the number of cycles the torsion bar will be subjected to as well as various operating conditions such as the loading conditions, the size, the finishing on the part, the operating temperature, and the desired reliability; the fatigue limit can be determined using the following equations:

$$S(N) = aN^b$$

Equation 9: Stress of Torsion Bar as a Function of Cycles

$$b = \frac{1}{z} log\left(\frac{S_m}{S_e}\right)$$

Equation 10: B value for Equation 9

$$z = log(N_1) - log(N_2)$$

Equation 11: z-value for Equation 10

$$log(a) = log(S_m) - 3b$$

Equation 12: a-value for Equation 9

Where N is the number of cycles to which the torsion bar is subjected, N_1 is always 1000 cycles, and N_2 represents the number of cycles at which the fatigue limit occurs. S_m is the mean strength at 10^3 cycles and S_e is the endurance limit of the material (for some materials this is called the fatigue limit and denoted as S_f); both the mean strength and the endurance limit can be calculated using equations 6.7 and 6.9 from Norton's Machine Design text, reproduced below:

$$S_m = 0.9S_{ut}$$

Equation 13: Mean Strength for Bending Loading

$$S_m = .75S_{ut}$$

Equation 14: Mean Strength for Axial Loading

$$S_e = C_{load}C_{size}C_{surf}C_{temp}C_{reliab}S_{e'}$$

Equation 15: Corrected Endurance Limit

Where S_{ut} is the ultimate tensile strength of the material; C_{load}, C_{size}, C_{surf}, C_{temp}, *and* C_{reliab} refer to constants determined by various conditions the torsion bar is subjected to, and are further defined in equation 6.7 of Norton's Machine Design text book. $S_{e'}$ refers to the estimated endurance limit and is determined for various materials using equation 6.5 of Norton's machine design text.

Using these equations we were able to develop spreadsheets allowing us to analyze the angle of twist, the torsion bar stress, and the safety factor for various diameters and lengths of the torsion bar. Using these spreadsheets we could easily alter the material properties to analyze different material options for the torsion bar. The materials we considered were low carbon stainless Steel, Aluminum 6065 and a series of plastics. The materials were chosen based on their yield strength, elastic modulus, cost and availability. The spreadsheets could then be compared to find the best diameter and length of the torsion bar, such that it would provide an appropriate angle of twist without being at risk of failure (examples of these spreadsheets can be found in Appendix L: Spreadsheet Analysis). The data points from the stress spreadsheet could then be used to generate a Goodman diagram to visualize the performance of each option, a Goodman diagram for aluminum 6061 can be seen below in Figure 14(for the complete set of Goodman diagrams produced please refer to Appendix A: Goodman Diagrams). After weeks of analysis it was concluded that the best option would be aluminum 6061 however this was not without serious compromise. By mounting the bar perpendicular to the direction of the bike, the bar was limited to be no longer than the length of the handle bars. This limitation restricted us to a maximum deflection angle of 9 degrees from a bar 10 inches in length and 1.25 inches in diameter. The team concluded that while potentially viable this option was poor due to the large length of the lever arm (approximately 8.5 feet for the torsion bar described above) required to achieve the necessary 16 inches of linear displacement to allow the pedals to complete a full cycle. With this decision the team came to the conclusion that the torsion bar would be a poor method of obtaining an assistive pedal force, and thus explored alternative options.

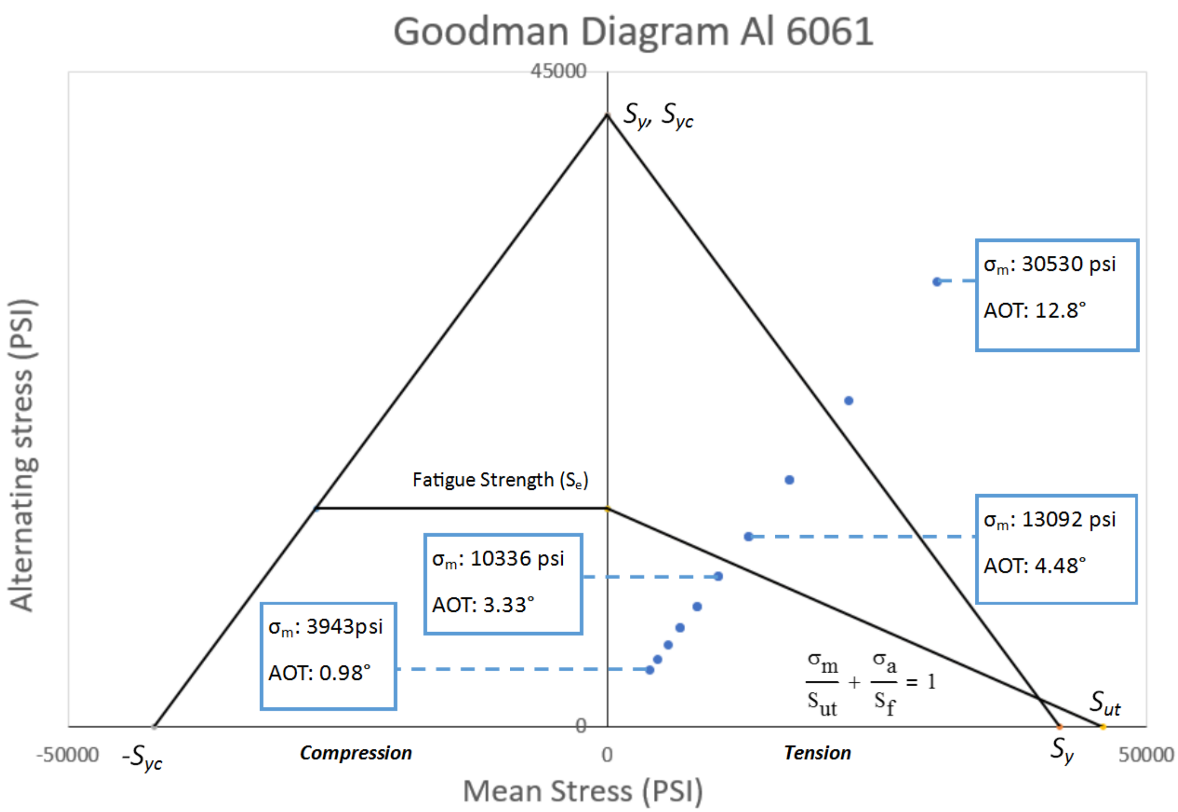

Figure 14: Goodman Diagram for Al-6061

With a far greater understanding of the problem at hand the team generated a new design of this retractor device, utilizing extension springs rather than a torsion bar or spiral springs to obtain the necessary pedal assistance force. In order to get proper linear deflection from the spring, the mounting placement was moved from behind the seat to under the top bar. Also a pulley is used to double the linear displacement of the extension spring to further our pursuit of 16in of displacement by the end of the steel wire that is attached to the pedal. Finally in order to pull the pedal up in the direction desired, a guidance pulley was added to the rear triangle of the bike directly above the axle of the back wheel, a depiction of this concept is seen below in Figure 15.

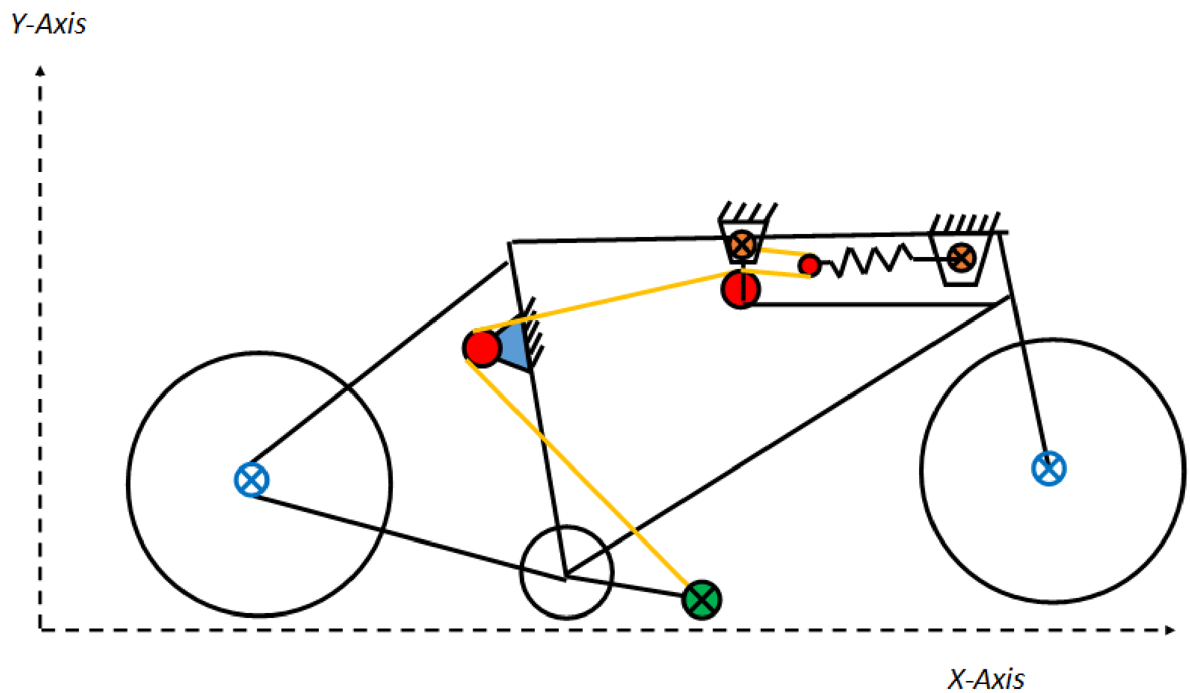

Figure 15: Representation of Spring Mechanism Design

After some brief research and analysis is was concluded that this embodiment of the retractor design would be the best way to meet our design specifications. More information of the final design of this concept will be included in the next section.

Final Design

The final design consisted of a force producing mechanism, consisting of the two extension springs and pulley system 1, from Figure 16 below, these components were contained between two U-channels, which were fastened to the top tube of the bicycle via aircraft clamps. A cable was routed through the pulley system, around a guidance pulley and attached to the pedal, to supply the necessary assistive force.

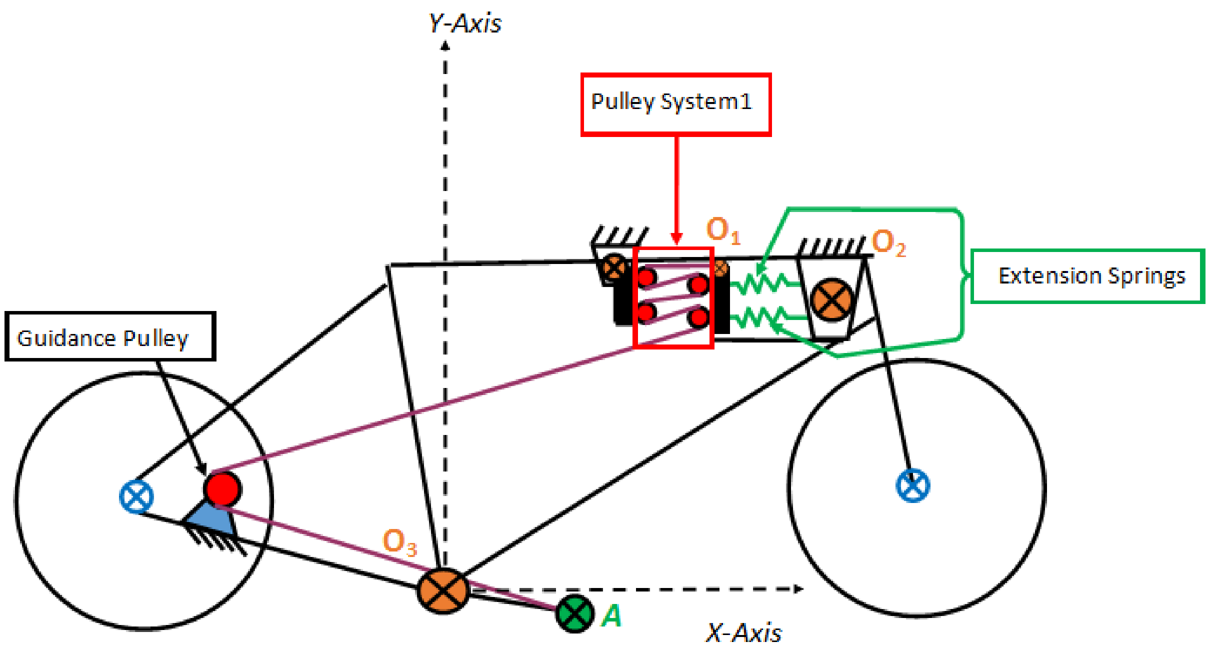

Figure 16: Depiction of Final Retractor Design

Linear Displacement

The bicycle crank is 8 inches from the center of its rotation axis to the center of the pedal. This means that during a full pedal cycle, the pedal experiences 16 inches of linear displacement. For the purposes of our device, this means the cable exiting the U-Channel will need to be capable of experiencing the required 16 inches of extension to account for the 16 inches of displacement at the pedal. The size limitations of the bike only provide 20 inches to place the spring mechanism in, since the springs have a free length of 8 inches, this leaves 12 inches of space for displacement. Since we needed to obtain 16 inches of displacement, we chose to utilize the mechanical advantage of a compound pulley system to increase the length of cable that exits the U-channel while decreasing the amount of displacement the springs will experience.

Pulley System

A compound pulley system provides mechanical advantage by dividing the amount of force, required to displace an object of weight (w), a distance x, by the number of rope passes (N) made through the pulley system, such that the user must pull the rope a distance of N times x to achieve an object displacement of distance x. For our design we need to increase the distance pulled to achieve the appropriate linear displacement of 16 inches. In order to maintain the required force output of the spring mechanism we will need to increase the spring rate by N times. Since we are now adding pulleys to our design the amount of room for linear displacement of the springs will decrease even further, the pulleys chosen have an outer sheave diameter of 2 inches, to make a compound pulley system we will need two of these pulley blocks, such that the amount of space allotted for linear displacement within the U-channel mechanism will be decreased by 4 inches from 12 inches to 8 inches. With the addition of pulleys to the mechanism we will need to include a method of mounting them, which will take up space as well. The mounting design for the pulleys takes away an additional 3 inches from our linear displacement available within the U-channel, this limits us to 5 inches of linear displacement. With 5 inches of available room for the springs to experience linear displacement, we will need a pulley setup that has at least 4 passes, which will require the springs to be displaced 4 inches to achieve the desired cable displacement of 16 inches.

Spring Selection

Discovered during analysis, the cable will need to be capable of exerting 30 lbs of pulling force on the pedal, this will provide the necessary torque about the crank axis to lift the dead weight of an average human leg (detailed analysis is provided in Appendix D: Torque Required to Lift Dead Weight of Leg and Appendix E: Pulley Design Analysis). Since we will be making 4 passes over the pulley system the force produced by the spring setup must be at least 120 lbs after the spring is displaced 4 inches (16/4 inches) which will require a spring rate of 32 lb/in. Due to size limitations for spring diameter created by the internal width of the U-channel being 1.25 inches, we chose to implement two springs in parallel, each with a spring rate approximately half of what is mentioned above. The spring rate of the springs we chose is 16 lb/in, with an initial tension of 8 lbs. Thus the combined spring rate is 32 lb/in with a total initial tension in the system of 16 lbs. At an extension of 4 inches, neglecting the initial tension these springs will produce a pulling force of 128 lbf, when we include the initial tension of 16 lbf, we have a total pulling force from the springs of 144 lbf which is only slightly more than the desired pulling force, but subsequently will be enough to lift the dead weight of a heavier than average leg.

Mounting Structure

For our design, we needed a way to enclose the device so that the mechanisms were at least partially covered and well supported. We also needed it to be light and easy to modify. We also have to consider ease of manufacturing and repairs. Our original design was to have one U-channel with a groove cut into the side. There would be a pin that slides in the groove to support the pulleys. We decided that this did not cover the mechanisms enough and that cutting a groove could be problematic. We decided to instead use two U-channels; one on top and one underneath. They would be spaced apart from each other to form a slot without the need for cutting one. The pulleys would also be supported more securely. In addition, the springs would be mostly covered up, making the device safer for the user. We wanted to minimize the weight of the channels, so we used 1/8th in thick aluminum channels that were 1 in wide with ½ in legs. The total length of each channel is 20 in.

In order to hold the channels together, we designed a set of brackets that hold the parts together. One would be used to mount the springs and be mounted to the back of the channels. The spring hooks would be attached to this bracket, via spring anchors. The bracket has legs on the sides so that bolts could be used to attach it to the channels, this bracket is depicted below in Figure 17. On the opposite end of the U-channel, there would be a similar bracket, used to mount the pulley, depicted in Figure 18.

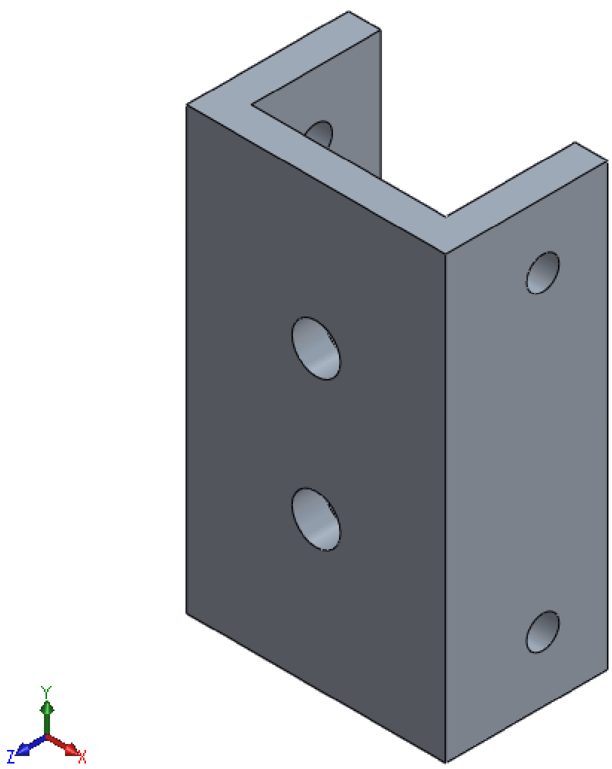

Figure 17: Spring Mounting Bracket

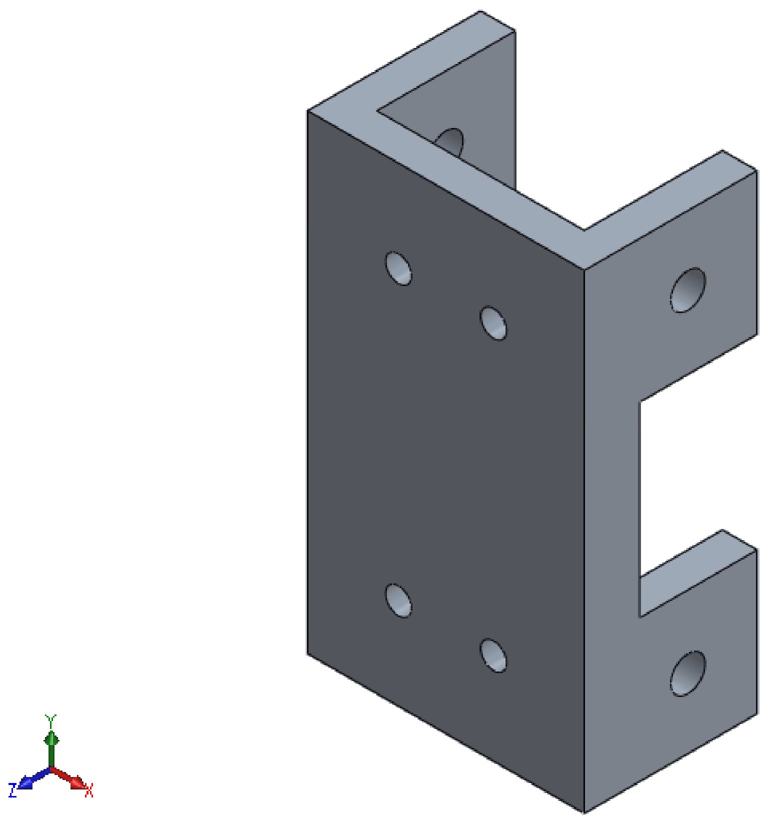

Figure 18: Pulley Mounting Bracket

In order to mount the device onto the bicycle, a series of airplane clamps were used. These are adjustable and easy to attach using just a screwdriver. To attach the clamps, a series of slots were cut into the top U-channel. The airplane clamps could then be fully opened and threaded through the slots. These aircraft clamps would also be used to hold the acrylic plate that supports the rear pulley in the proper position.

Guidance Pulley

In order for the device to complete the task of lifting the rider's leg, the energy from the springs and the displacement of the wire must be directed effectively. The force from the springs must be supplied to the pedal at the correct position and from an ideal angle. The wire must also not interfere with the normal motion of the rider’s leg. To do this a method of guiding the wire along its path was designed.

The solution that was chosen to guide the wire was a side mounted pulley that would change the direction of the wire from its position where it exits the device. The hardware chosen was a side mounted single pulley pictured below in Figure 19. This part is very thin so the wire is kept close to the bike. This is advantageous as compared block pulley which would pull the wire away from the frame which brings up issues with safety and would result in a loss of force exerted on the pedal due to the transmission angle of the wire.

Figure 19: Side Mount Guidance Pulley

The challenge of designing this aspect of the final device was choosing a mounting mechanism. Our key requirements were that the position of the pulley would be such that when the pedal reached the bottom part of the pedal stroke the potential energy in the springs would be released and that there would not be a significant loss in the transfer of energy due to a large transmission angle. After conducting a force analysis (Appendix E: Pulley Design Analysis), a position inside the rear triangle of the bike was determined. This position is both (a) above the pedal so it will begin pulling on the pedal after it reaches the bottom of the stroke and (b) results in the best possible angle of transmission, which is tangential to pedal direction, during most of the weak portion of the rider's pedal stroke. In order to mount the pulley in this rear section of the rear triangle a mounting plate was designed, displayed below in Figure 20. This plate was meant to use aircraft clamps to hold it to the frame of the bike and the guidance pulley would be bolted to the plate. The plate was designed out of acrylic because of its light weight, low cost, and manufacturing ease. A tear out analysis was completed to ensure the material would be strong enough to hold up to the forces applied both at the pulley and the clamps (detailed calculations of this can be found in Appendix G: Guidance Pulley Mounting Plate Tear-Out Analysis). The final design utilizes the least material possible while still allowing for some aesthetic additions to the plate.

Figure 20: Mounting Plate for Guidance Pulley

Pedal Connection

The final step in our design was to connect the wire to the pedal. The simple solution we chose was to loop the wire around the connector between the pedal and the crank shaft. To do this a steel wire rope thimble clip was purchased along with some U bolts made for the wire diameter chosen. The thimble clip is used to keep the loop of wire around the pedal large enough to not do damage to the wire and to prevent the wire from sliding up the shaft. The U bolts ensure that the wire stays looped around the pedal and does not come undone during usage of the device.

Figure 21: Thimble Clip, Used to Attach Cable to Pedal

The final spring mechanism is depicted below in Figure 22.

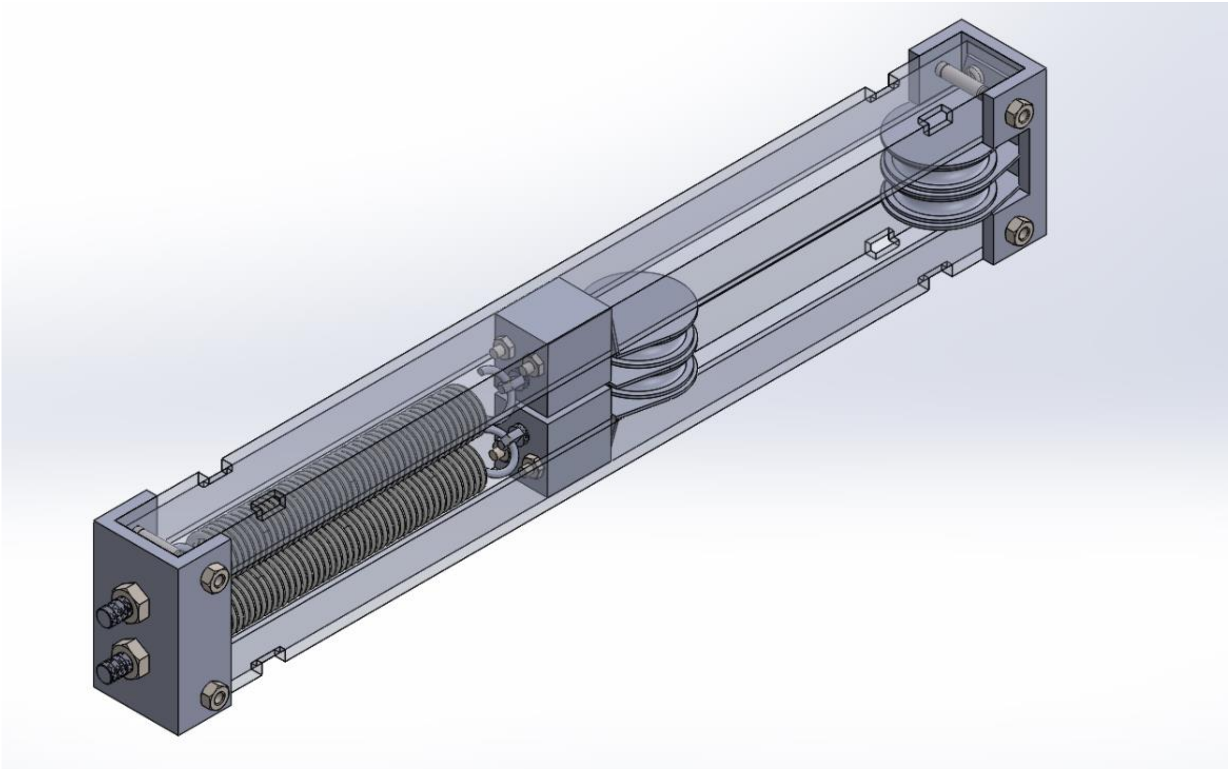

Figure 22: Final CAD Model of Spring Mechanism

Results

During assembly of the spring mechanism, the team was experimenting with sliding the acrylic block back and forth within the U-channel, in doing this we discovered that there was a considerable amount of friction present between the sliding blocks and the aluminum U-channels. We became concerned that this large amount of friction would cause issues when the device was completed. The input force required by the user felt excessive during the trials.

When initially attaching the device to the bike, we were short one aircraft clamp that was needed to attach the acrylic plate to the bike. We decided to attach the guidance pulley mounting plate to the bike with only three aircraft clamps, rather than the four we had designed it to use. Unfortunately, due to a decreased shearing area, caused by the missing aircraft clamp, the stress induced by the operation of the spring mechanism caused the aircraft clamp, to tear out of the plate; luckily we had manufactured a spare.

After breaking our first guidance pulley mounting plate, we acquired the missing aircraft clamp, and attached the alternate mounting plate to the bike using all four aircraft clamps as we originally intended. As our analysis had predicted, the part would not fail when correctly attached to the bike. With the guidance pulley properly attached and the device working we began judging how well the device worked. The first thing we noticed during our tests was that the guidance pulley had been mounted too low; making it so the device would provide an assisting force between about 90 and 270 degrees of the crank cycle. We had wanted an assisting force between the crank angles of about 170 and 350 degrees, with the crank angle range needing the most assistance being 180 to 245 degrees. By choosing to mount the guidance pulley higher we could have achieved a resulting crank angle of assistance closer to that which we desired. This issue with location was a result of using a different bicycle than we had originally designed our system for.

The second largest issue we discovered about our device was how much force it required to extend the spring system during the downward pedal stroke. With both springs in place it felt as though the springs pulled the pedal through its pedal stroke with an acceleration that may be uncomfortable or unsafe for a person suffering from diminished muscular strength. As an experiment we disconnected one of the springs, cutting the output force in half, while this required a much more reasonable force to extend the springs the team didn't feel like the device was providing quite enough of an assisting force when the spring retracted; however, through discussion our team determined that the lower acceleration of the return pedal stroke from the use of one spring was more ideal for the devices intended use.

The final and least worrisome issue discovered in the device, was that the mechanism could not fit completely in the frame of the bike, as we had hoped. This was due to the size of the pulley blocks, with pulley blocks that were just slightly too wide to fit the pulley vertically aligned in the slot of the U-channel, we were forced to mount the pulleys in a horizontal alignment. The horizontal mounting orientation of the pulley required the two U-channels to be separated with a larger gap than a vertical pulley orientation would have required, resulting in a final mechanism height of 3.40 inches. With a head tube length of 3 inches, we could not make use of the full length of the top bar, as we had intended to. This forced us to mount the device with a diagonal offset from the center of the frame. With the spring mechanism mounted at an angle, the cable exiting the spring mechanism was forced to rub against the edge of the aluminum spring mounting bracket, as it made its path to the guidance pulley; this rubbing caused, the cable to abrade its protective plastic coating, and the team postulated that eventually the cable would fray and subsequently snap after repeated uses.

Conclusions

The goal statement of this project was to design and build a device to enable a user who has reduced usage of the hamstring to ride a bicycle by aiding leg motion between the pedal angles of 180° to 225°. Considering this, the device produced comes very close to meeting this goal, but still requires improvements and further prototyping in order to fully reach the goal.

The final design does unquestionably aid leg motion through the angles specified, but it does so with some very significant drawbacks. The greatest of these is the large input force required from the rider during the down-stroke in order to extend the springs. By demanding so much energy from the rider during this phase there will be greater fatigue while riding and reduced energy for propulsion. Furthermore, given that this device is meant for a stroke victim the input force required might be more than they can exert. If a rider was unable to extend the spring then this device would only be successful in moving the issue of not being able to complete a pedal stroke to a different phase of the pedal stroke. This concern makes it very questionable whether this device would be helpful to a stroke victim. However, as stated in the results section it is possible that only using one spring would be more effective and could possibly alleviate this concern.

Solving the issues with this device may not be as simple as just removing a spring. Our calculations and tests show that it takes more force than that spring can supply to lift and average man’s leg. This device may require future engineers to look into more advanced solutions to supply forces as the current device would not be comfortable to ride even for an able bodied person. The device may need to employ clever mechanisms that utilize mechanical advantage or electromechanical devices to generate a truly useful device. In the current prototype the high input force required and the loss of energy when it is finally released both indicate the device may need more elaborate methods of mechanical advantage in order to make it usable for a stroke victim. A potential stronger solution to the issue would be the use of an electric motor. This would significantly drop the devices reliance on the rider and could remove the need to for complex mechanisms which could pose safety risks to the rider. Much more prototyping would need to be done to explore these concepts.

The greatest challenges that the team faced when designing this device was likely the design requirements we had placed on ourselves. Demanding that the device be purely mechanical and easily attachable to many bike was a demanding pair of requirements that significantly limited our imagination. While such a device would be both novel and ideal, it is doubtful that such a device would be the most effective way to aid a stroke victim in riding a bicycle. This team concluded after our attempt at creating such a device that a broader approach to solving the problem with more succinct design specifications should be used by engineers exploring this concept in the future. Some of these possible approaches will be discussed in the next and final section of this paper.

Recommendations and Future Work

After testing our prototype, we concluded that certain aspects of the design could be improved. Our original prototype had issues fitting onto the bicycle, and with the amount of force required by the user. We found out that our original assumptions used to calculate the necessary force were wrong, and that the required force was actually less than we had expected.

To address these concerns, certain mechanical changes to the device are possible. First, the friction of the device could be reduced in the slider block through the use of a layer of low-friction material, or changing the block material to a low friction material. The force could also be reduced by using a single custom spring that provides a more suitable force. In addition to being reduced, the force needs to be applied at the ideal portion of the pedal stroke. This problem arose when we tested the bicycle on a different bicycle than we had originally designed the device for. This could be accomplished by adjusting the location of the guidance pulley.

To make the device fit properly in the bicycle, it must be reduced in size. By using smaller pulleys in addition to the single custom spring the distance between the two U-channels could be reduced. This would reduce the total height of the device, making it fit better in the available space. It also has the added benefit of covering the mechanisms, making the device safer for the user.

Other improvements to the device go beyond just re-arranging the components. Future work could include an analysis of potential materials to find ones more suited to the desired goal. This would be an improvement over just using materials that were chosen based on cost and availability. Different designs could also be considered. For example, the use of a motor to apply force would not increase the force required from the user at the non-powered portion of the pedal stroke. Another design would be to use a flywheel to store the energy. The wheel could be precharged with a motor in a charging stand. This design would have the advantage of using an external power source without requiring the user to carry the weight of the motor on the bicycle. A more complicated system could be redesigning a regenerative braking system, to harness the energy of the spinning wheel and transmit it to the pedals.

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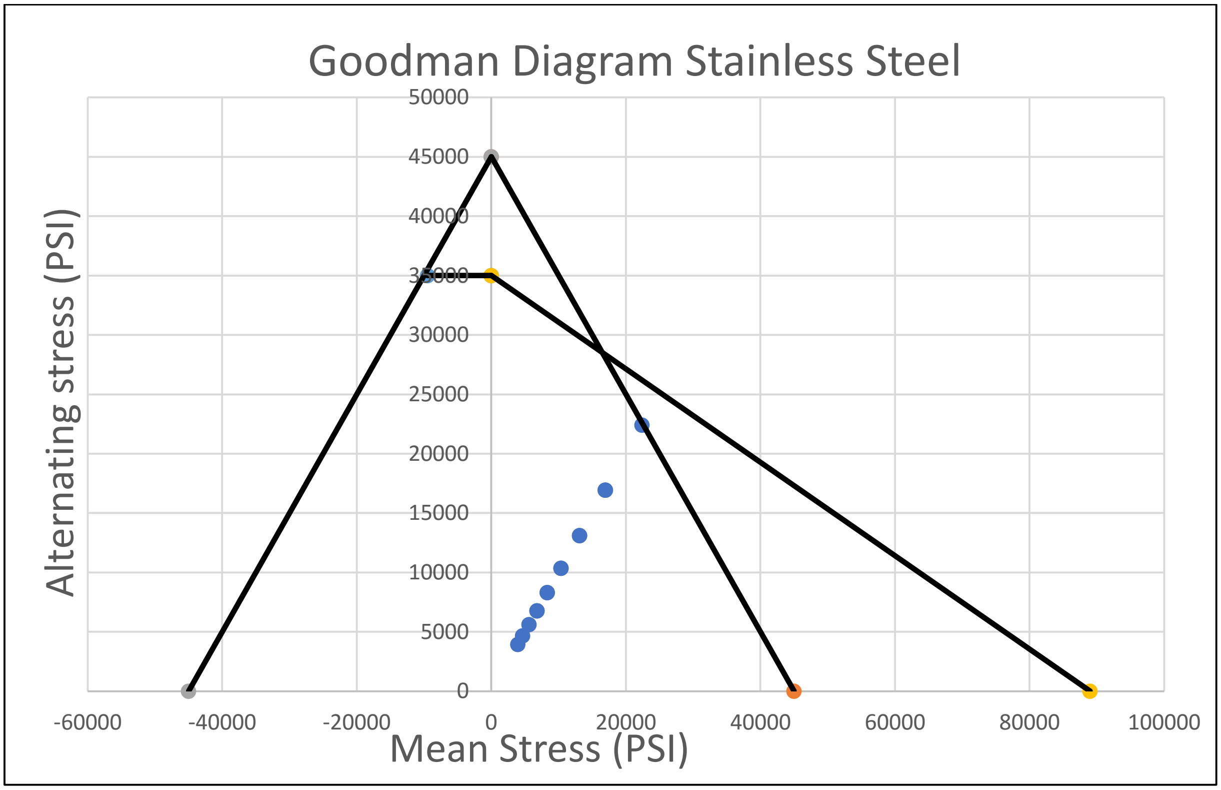
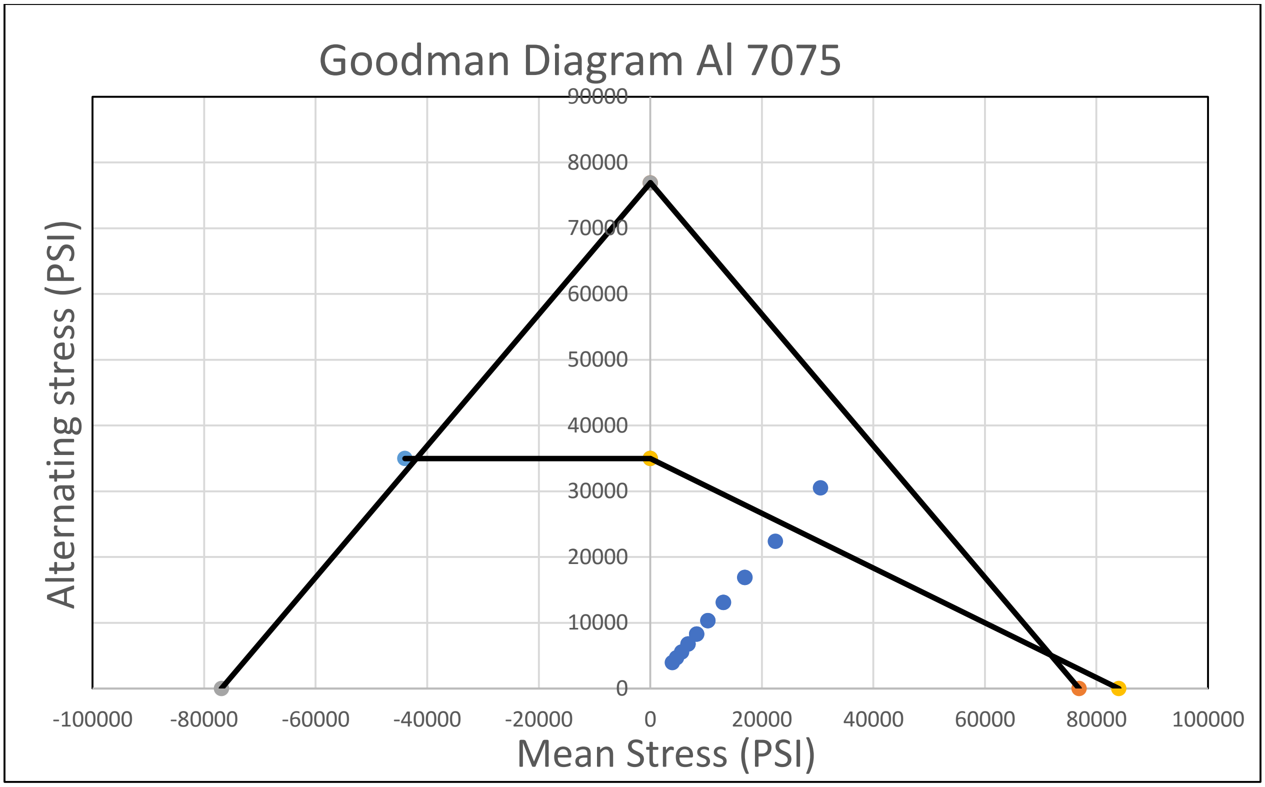
Appendix A: Goodman Diagrams




Goodman Diagram Al 7075
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70000
60000
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20000
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0
20000
40000
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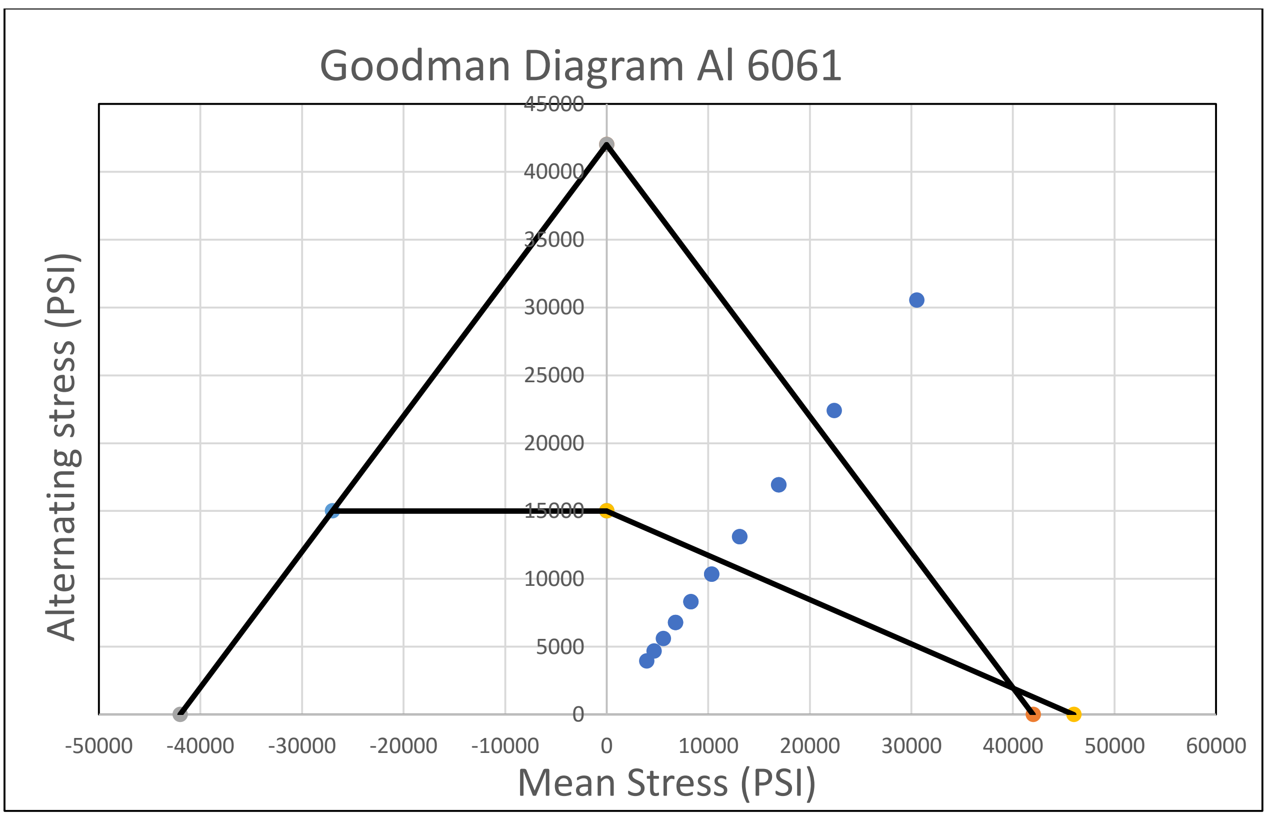
80000
100000
Mean Stress (PSI)
Alternating stress (PSI)


Goodman Diagram Al 6061
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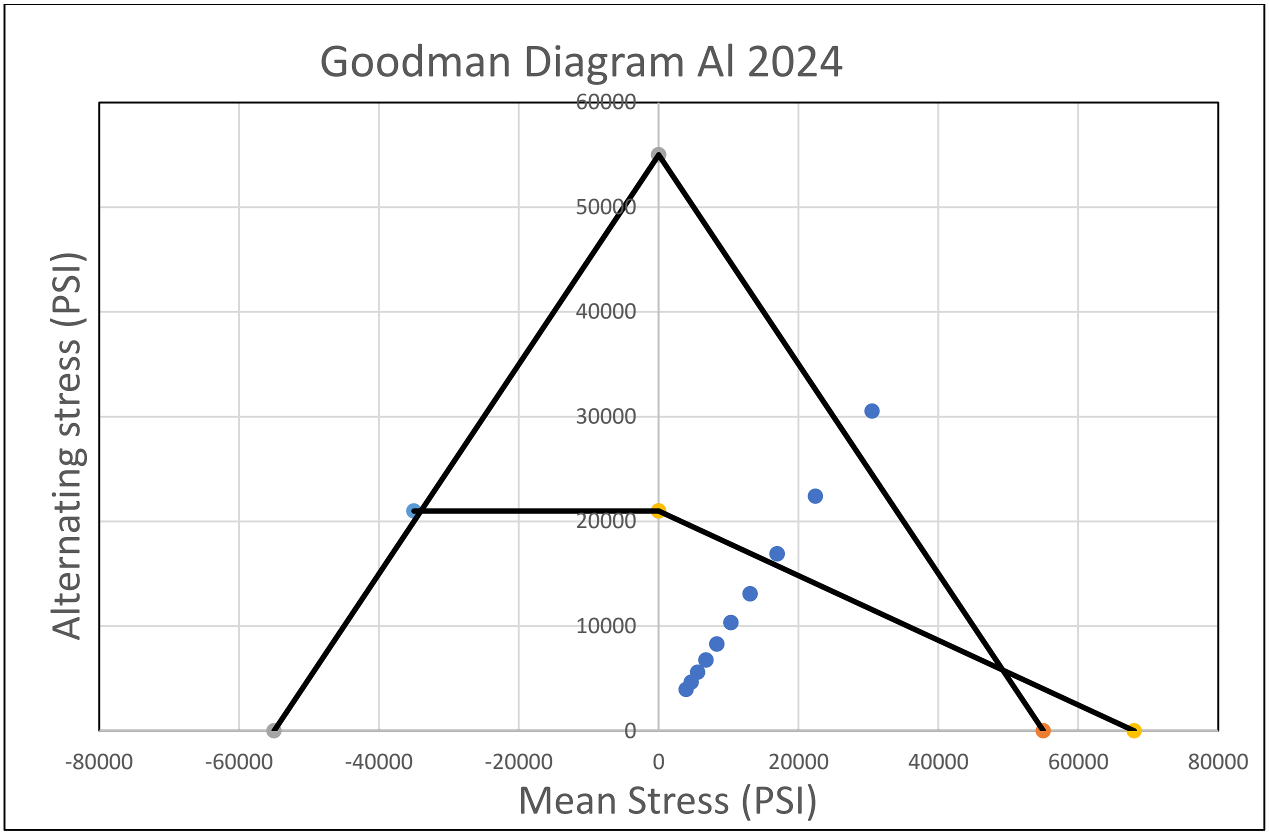
Mean Stress (PSI)
Alternating stress (PSI)


Goodman Diagram Al 2024
Alternating stress (PSI)
Mean Stress (PSI)
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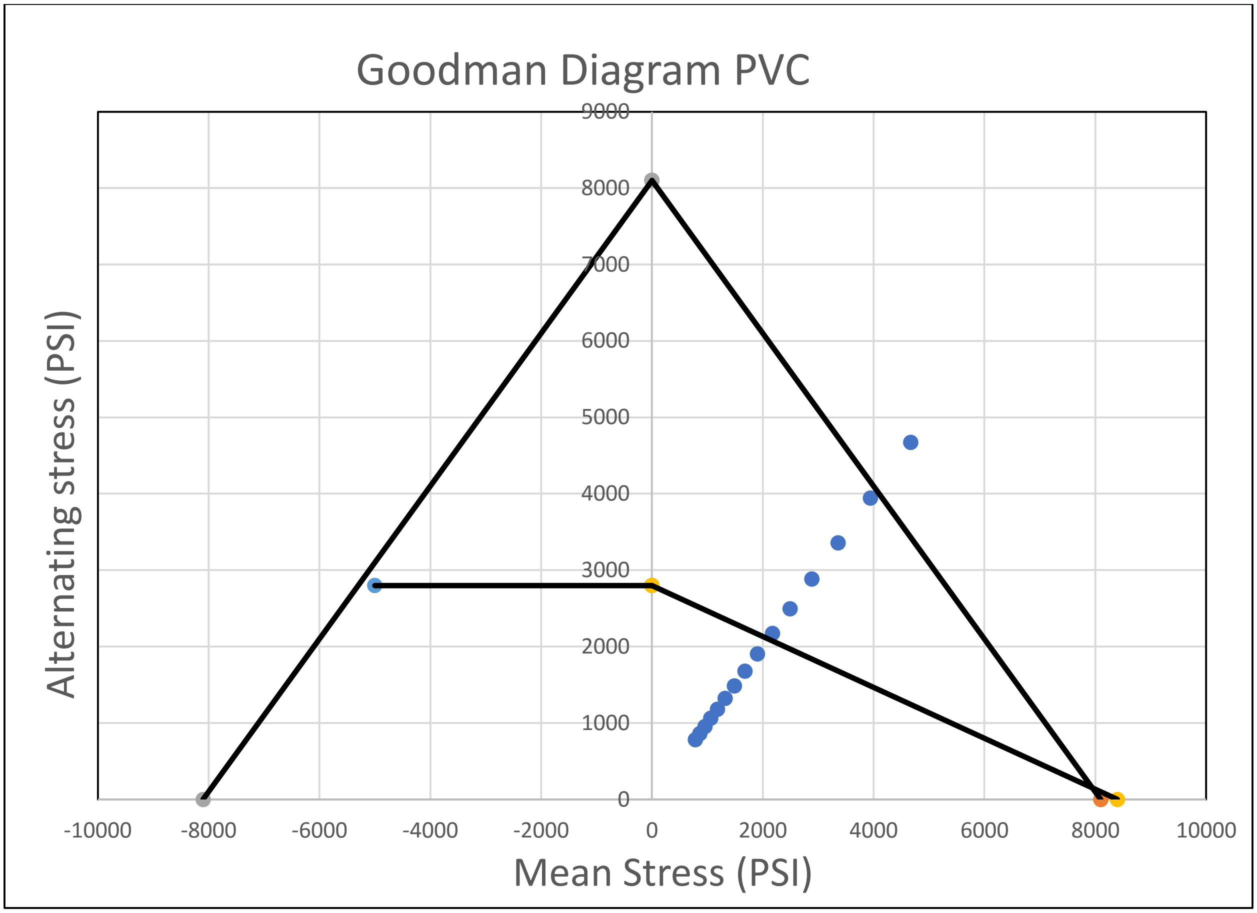
20000
40000
60000
80000


Goodman Diagram PVC
Alternating stress (PSI)
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4000
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0
2000
4000
6000
8000
10000
Mean Stress (PSI)

Appendix B: Force Required to Accelerate Bike

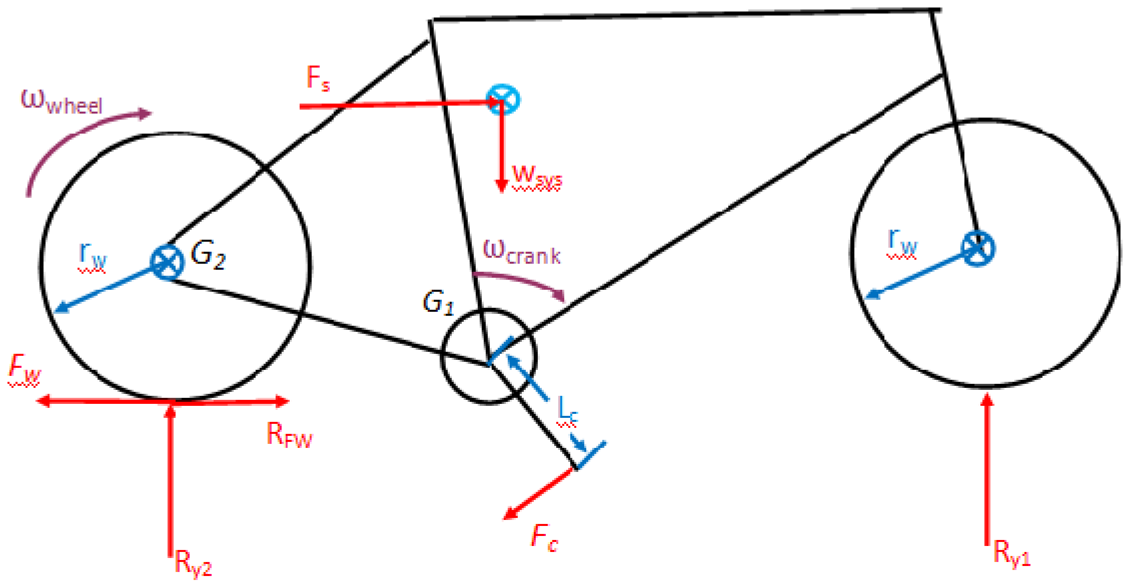


Figure 1: Free Body Diagram of Bicycle

The following calculation will determine the necisary force F_c required to be applied at 90 degrees to the crank arm, to achieve a desired acceleration as outlined bellow. The desired acceleration was determined ***through the use of table 7-1 found in Norton Design of machinery,*** the value for a normal vehicle acceleration was used.

Where: L_c is the length of the crank

r_w is the radius of the wheel

f is the friction coefficient between the wheel and the ground

w_{bike} is the weight of the bike

w_{person} is the weight of the person riding the bike

w_{device} is the weight of the assisting device

w_{sys} is the combined weight of the system

m_{sys} is the mass of the system

GIVENS::

$L_c := 8in$ $\quad r_w := 13in$ $\quad w_{bike} := 30lbf$ $\quad w_{person} := 150lbf$ $\quad w_{device} := 5lbf$

$f := .75$ $\quad w_{sys} := w_{bike} + w_{person} + w_{device} = 185 \cdot lbf$

$$m_{sys} := \frac{w_{sys}}{g} = 185\,lb$$

KNOWN//ASSUMED VARIABLES::

$G_1 := 4$ $\quad G_2 := 6$

$$a := .1 \cdot g = 3.217 \frac{ft}{s^2}$$

Where: G_1 is the diameter of the driving gear, attached to the crank
G_2 is the diameter of the driven gear, attached to the wheel
ω_{crank} is the assumed average rotational velocity of the crank
a is the desired acceleration of the system

CALCULATIONS::

$F_s := m_{sys} \cdot a = 18.5 \cdot lbf$

F_s is the force required to propell the system mass at the desired acceleration

The required reaction force at the wheel (R_{FW}) is equal to the force required to propell the system (F_s). Since the wheel has a friction coefficient less than 1 some of the force produced by the wheel will be lost, thus the wheel must produce a slightly higher force to account for this. Calculating for the output force of the wheel to achieve the desired R_{FW} we have:

$$F_w := \frac{F_s}{f} = 24.667 \cdot lbf \qquad f = 0.75$$

The required force applied 90 degrees to the crank will be dependent on the gear ratio the rider has selected, for our purposes we have assumed the highest gear ratio (biggest gear at the cranks, and smallest gear at the wheel). Calculating for this force we have:

$$F_c := \frac{F_w \cdot r_w \cdot \left(\frac{G_1}{G_2}\right)}{L_c} = 26.722 \cdot \mathrm{lbf}$$

F_c is the force required to be applied at 90 degrees to crank, to achieve desired acceleration, dependent on the gear ratio.

We can use this force to determine the appropriate spring constants, and deflections to be used within our design such that we ensure our design will provide the desired acceleration.

Appendix C: Four-Bar Analysis and Torsion Bar Calculations

KNOWN: Provided with the required force to obtain a desired acceleration (as calculated previously in another document), the length of the bycicle crank and the length of the lever arm used to apply torque to a torsion bar.

FIND:

1.) The required length for the connecting rod, the link connecting the crank are to the lever arm.

2.) Determine the angles θ_2, θ_3, θ_4 as functions of the crank angle θ_c

    2A.) Determine the transmission angle μ

3.) Determine the force acting along link 3 as a function of the required tangential force F_t (previously calculated)

    3A.) Determine the maximum and minum force acting along link 3, and the corresponding crank angle at which these extremes occur

4.) Determine the required angular deflection of the torsion bar to achieve 16 in of linear displacement at the end of the desired lever arm length

5.) Use information found in parts 1-4 combined with an assumed torsion bar length to determine the required diameters of a torsion bar, for a solid circular cross section, and a hollow closed circular cross section.

6.) Use information calculated in part 5 to determine the weight of each torsion bar design

SCHEMATIC: for geometrical analysis:

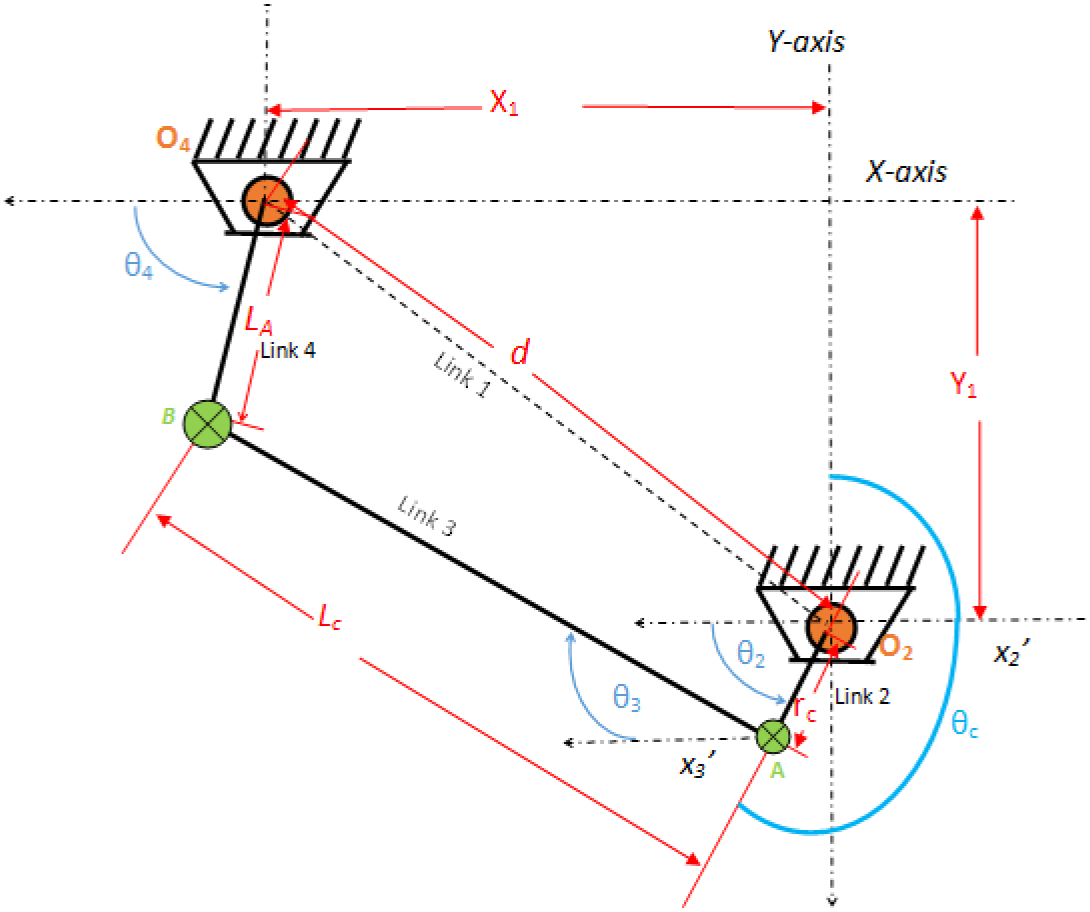


Figure 1: system Device Modeled as a 4-bar crank-rocker mechanism in an open configuration

Where:

r_c is the length of the crank on the bike and represents link 2 in a fourbar mechanism

L_c is the length of the connecting rod and represents link 3 in a fourbar mechanism

L_A is the length of the leverarm attatched to the torsion bar and represents link 4 in a fourbar mechanism

d is the shortest distance between the the origins O_2 and O_4 and represents link 1 of a fourbar mechanism

θ_c is the angle of the crank arm, in relation to the y-axis, as depectited in figure 1

θ_2 is the angle between the positive x_2' axis and the crank arm (link 2)

θ_3 is the angle between the positive x_3' axis and the connecting rod (link 3)

θ_4 is the angle between the positive X-axis and the lever arm (link 4)

X_1 represents the distance in the horizontal direction between O_2 and O_4

Y_1 represents the distance in the vertical direction between O_2 and O_4

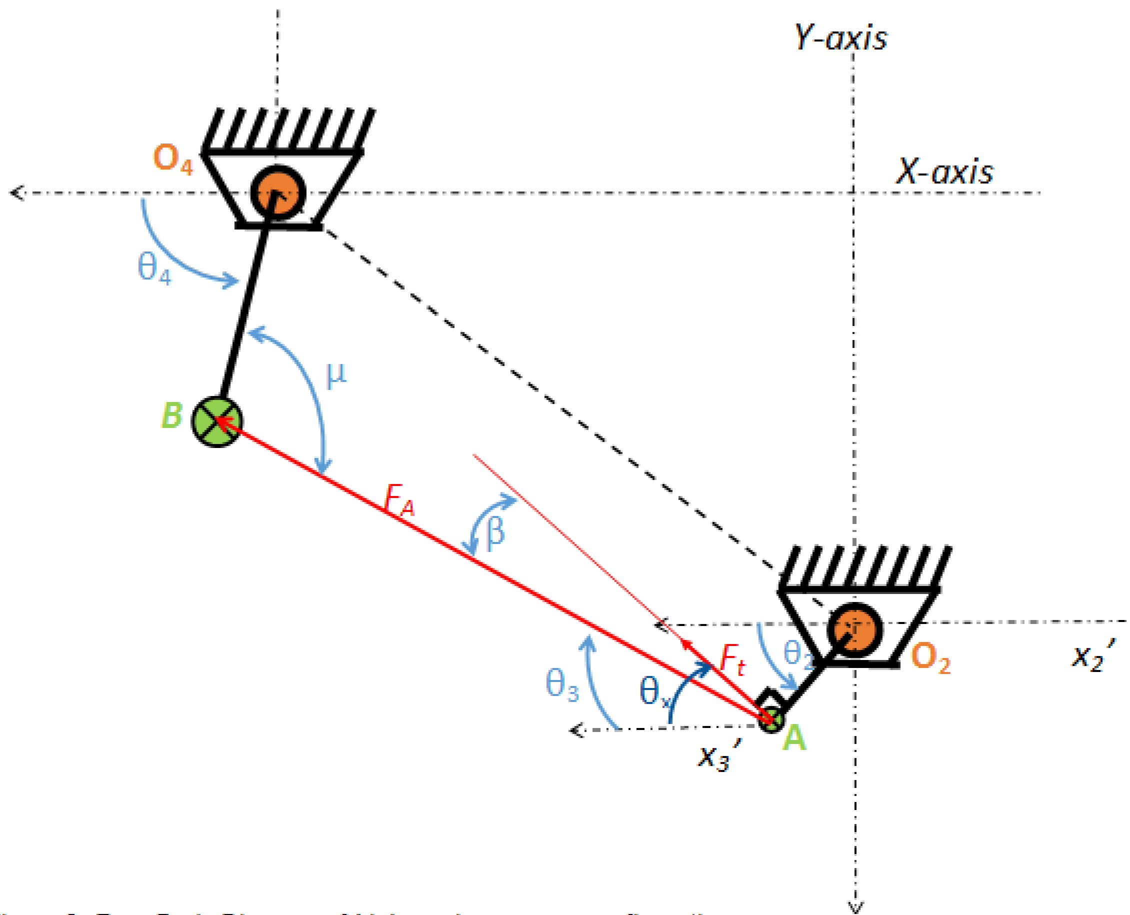


Figure 2: Free Body Diagram of Linkage in an open configuration

Where:

F_t is the tangential force required to act @90 degrees to the crank to achieve the desired acceleration

θ_x is the angle between the x_3' axis and F_t

F_A is the force that can be applied through the connecting rod (link 3) and delivered to the lever arm (link 4) attached to the torsion bar (O_4)

μ is the angle between the connecting rod and the leverarm (link 3 and link 4 respectivly), and is refered to as the transmission angle

β is the angle between F_t and F_A

GIVEN DATA: for geometrical analysis:

DRIVING VARIABLES::

$r_c := 8in$ $\quad L_A := 18in$ $\quad X_1 := 30in$ $\quad Y_1 := 28in$ $\quad F_t := \frac{70}{2}lbf$

$$\theta c := 0..360$$

$$\theta_{\theta c} := \frac{2\pi}{360}\theta c$$

DRIVEN VARIABLES::

$\theta_2(\theta_c) := 270deg - \theta_c$ $\quad d := \sqrt{X_1^{\,2} + Y_1^{\,2}} = 41.037 \cdot in$ $\quad L_c := \sqrt{(Y_1 - L_A + r_c)^2 + X_1^{\,2}} = 34.986 \cdot in$

EQUATIONS REQUIRED FOR FOUR BAR ANALISYS:

$A_x(\theta_c) := r_c \cdot \cos(\theta_2(\theta_c))$ $\quad A_y(\theta_c) := r_c \cdot \sin(\theta_2(\theta_c))$

$$S(\theta_c) := \frac{r_c^{\,2} - L_c^{\,2} + L_A^{\,2} - d^2}{2 \cdot (A_x(\theta_c) - d)} \qquad P(\theta_c) := \frac{A_y(\theta_c)^2}{(A_x(\theta_c) - d)^2} + 1$$

$$Q(\theta_c) := \frac{2 \cdot A_y(\theta_c) \cdot (d - S(\theta_c))}{A_x(\theta_c) - d} \qquad R(\theta_c) := (d - S(\theta_c))^2 - L_A^{\,2}$$

ANALISYS OF A FOUR BAR MECHANISM IN AN OPEN CONFIGURATION:

$$B_y(\theta_c) := \frac{-Q(\theta_c) - \sqrt{Q(\theta_c)^2 - 4 \cdot P(\theta_c) \cdot R(\theta_c)}}{2 \cdot P(\theta_c)}$$

$$B_x(\theta_c) := S(\theta_c) - \frac{2 \cdot A_y(\theta_c) \cdot B_y(\theta_c)}{2 \cdot (A_x(\theta_c) - d)}$$

$$\theta_3(\theta_c) := \operatorname{atan}\left(\frac{B_y(\theta_c) - A_y(\theta_c)}{B_x(\theta_c) - A_x(\theta_c)}\right)$$

$$\theta_4(\theta_c) := \operatorname{atan}\left(\frac{B_y(\theta_c)}{B_x(\theta_c) - d}\right)$$

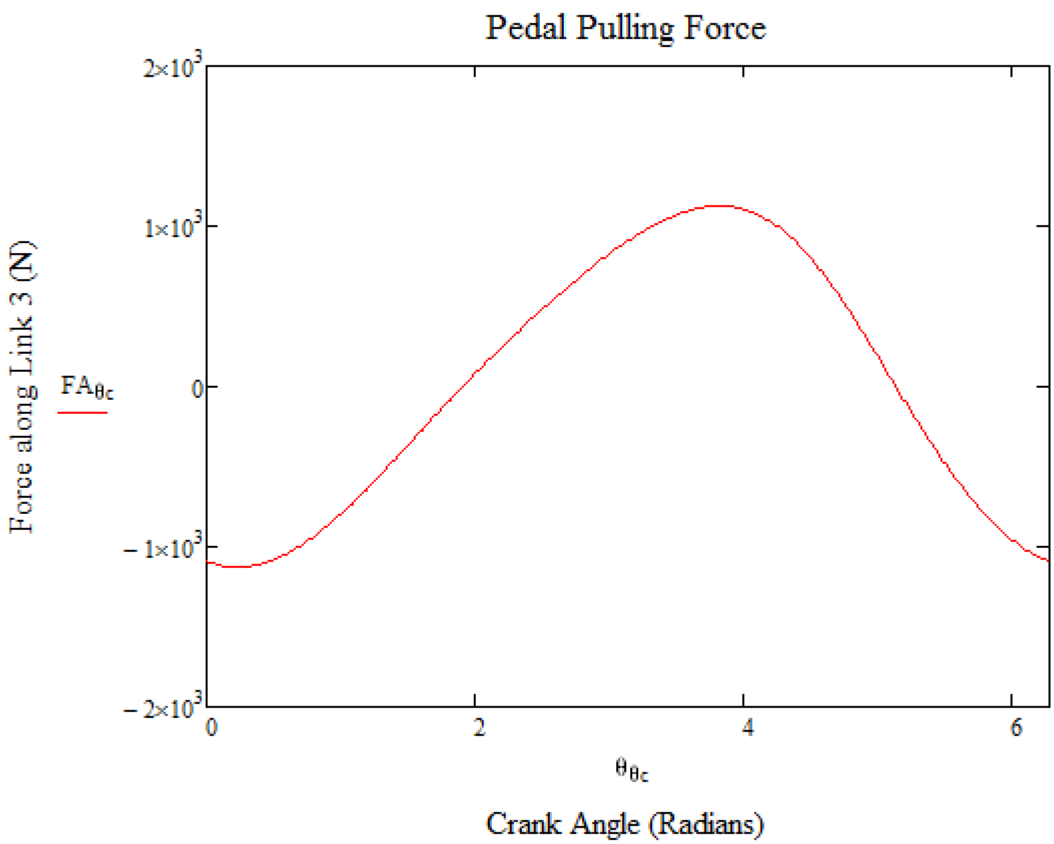
$$\theta_x(\theta_c) := (180\text{deg} - \theta_2(\theta_c)) - 90\text{deg}$$

$$\mu(\theta_c) := \left|\theta_3(\theta_c) - \theta_4(\theta_c)\right|$$

$$\beta_2(\theta_c) := \left|\theta_x(\theta_c) - \left|\theta_3(\theta_c)\right|\right|$$

$$FA_{\theta c} := F_t \cdot \cos(\beta_2(\theta_{\theta c}))$$

these calculations for F_A show the amount of Force that will need to be transmitted along the conncecting rod (L_c from Figure 1.) inorder to generate the F_t, such that the system is capable of achieving the desired acceleration.



Plot 1: The applied force F_A acting through link 3, plotted against the crank angle θ_c

$F_{max} := \max(FA) = 35\,\mathrm{lbf}$

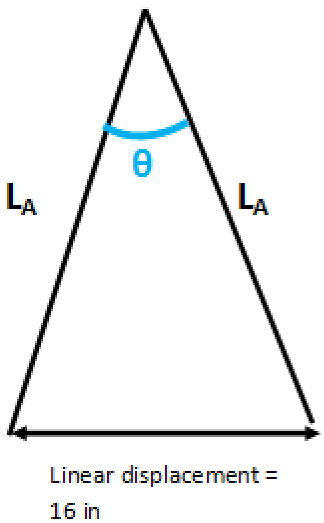
$\theta_{cMaxF} := \mathrm{match}(\max(FA), FA) = (219)$

$F_{min} := \min(FA) = -35\,\mathrm{lbf}$

$\theta_{cMinF} := \mathrm{match}(\min(FA), FA) = (13)$

the max and min functions highlighted in light blue return the maximum and minum forces required through 1 complete rotation of the crank. The coresponding match functions, highlighted in orange, return the index at which these values are found, in this case the index value coresponds to the crank angle θ_c

Calculating the required angular deflection of the torsion bar to achieve the necisary linear displacement of 16in



L_d is the required linear displacement at the end of the lever arm, this is driven by the length of the crank.

$$L_d := 16\text{in}$$

Figure 3: Schematic for determining torsion bar angular deflection θ, based on linear displacement

using the law of cosines to calculate for θ given a pre-determined lever arm length we have:

$$\theta := \text{acos}\left(\frac{L_A^2 + L_A^2 - L_d^2}{2 \cdot L_A \cdot L_A}\right) = 52.776 \cdot \text{deg}$$

using the law of cosines to calculate the length of the lever arm (L_1) given an angular deflection for the torsion bar

$$L_{arm} := 22\text{in}$$

$$\theta_{torsionBar} := 4.33\text{deg}$$

Given

$$\cos(\theta_{torsionBar}) = \frac{L_{arm}^2 + L_{arm}^2 - L_d^2}{2 \cdot L_{arm} \cdot L_{arm}}$$

$$\text{Find}(L_{arm}) = 17.647\,\text{ft}$$

TORSION BAR CALCULATIONS:

KNOWN: The transmission angle μ as calculted above, the force applied through the connecting link (link 3) F_A as calculated above. The modulous of rigidity for the proposed material to be used for the torsion bar. The cross sectional geometry of the torsion bar (circular, as outlined above). The desired angular deflection of the torsion bar θ. The equation for angular deflection of a torsion bar.

ASSUME: a length for the torsion bar

FIND: The required diameters of torsion bar, D for a closed circular cross section; D_o , d_i for a tubular cross section.

SCHEMATICS:

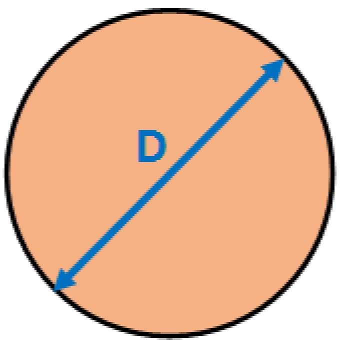


Figure 4: Cross Section of Closed Circular Torsion Bar

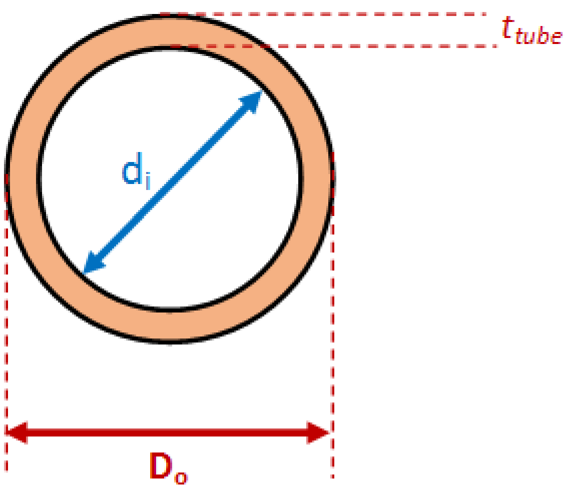


Figure 5: Cross Section of Tubular Torsion Bar

EQUATION FOR ANGULAR DEFLECTION OF A TORSION BAR::

$$\theta = \frac{T \cdot l}{J \cdot G}$$

Where::

θ = the angular deflection of the torsion bar, for the given lever arm this has been calculated to need to be 54 degrees

T = the torque applied to the torsion bar

l = the length of the torsion bar from its grounding point to location of applied torque

J = the polar moment of area

G = modulous of rigidity (a material property)

VARIABLES:

DEFINED:

The shear modulous of rigidity as obtained from Aerospace Specification Metals Inc. for AISI type 304 Stainless Steel is:

$$G_{st304} := 86\text{GPa}$$

KNOWN:

$$D := \frac{3}{16}\text{in}$$

UNDEFINED--VARIABLE GUESSES:

$$l_c := 6\text{in}$$

EQUATIONS:

$$J_0 := \frac{\pi \cdot D^4}{32}$$

The torque applied to the torsion bar will be the product of the force acting perpendicular to the leverarm (link 4), and the length of the lever arm. Thus we have:

$$F_{TLA} := F_{max} \cdot \sin\left(\mu\left(\theta_{cMaxF}\right)\right) = 33.859\,\text{lbf}$$

$$L_A = 18 \cdot \text{in}$$

$$T_1 := L_A \cdot F_{TLA} = 609.459 \cdot \text{lbf} \cdot \text{in}$$

Given

$$\theta = \frac{T_1 \cdot l_c}{\frac{\pi \cdot D^4}{32} \cdot G_{st304}}$$

$$L_0 := \mathrm{Find}(l_c) = 2.287 \cdot in$$

$$L_1 := \left|(L_0)\right| = 2.287 \cdot in$$

Where L_1 is the recomended length of the torsion bar, for the required torque T_1 at the given transmission angle μ.

Tubular Cross Section Torsion Bar Calculations:

The only difference in the calculation of angular deflection for a tubular torsion bar is the equation used for the polar moment of area (J); for a tubular cross section this is, $J_t := \frac{\pi \cdot \left(D_o^{\ 4} - d_i^{\ 4}\right)}{32}$

Solving for d_i of a tubular cross section we have:

ASSUMED VARIABLE VALUES:

$$D_o := \frac{1}{4} in \qquad d_i := 0.1940 in$$

D_o is the desired outer diameter of the tubular torsion bar, l_t represents the length of the tubular cross section torsion bar.

undefined guess value for the inner diameter of the tubular torsion bar:

$$l_t := 3in$$

Given

$$\theta = \frac{T_1 \cdot l_t}{\frac{\pi \cdot \left(D_o^{\ 4} - d_i^{\ 4}\right)}{32} \cdot G_{st304}}$$

$$L_2 := \mathrm{Find}(l_t) = 4.608 \cdot in$$

$$L_3 := \left|(L_2)\right| = 4.608 \cdot in$$

$$t_{tube} := (D_o - d_i) = 0.056 \cdot in$$

where t_{tube} represents the wall thickness of the tube, and L_3 represents the required length of the torsion bar to achieve the desired angular deflection.

Weight Calculations:

Using the information calculated above combined with material property data we can now find the difference in weight between the two torsion bar cross sections.

The weight density as obtained from Aerospace Specification Metals Inc. for AISI type 304 Stainless Steel is:

$$\gamma_{st304} := 0.289 \frac{lbf}{in^3}$$

Closed Circular Cross Section:

$$A_c := \frac{\pi \cdot D^2}{4} = 0.028 \cdot in^2$$

$$V_c := A_c \cdot L_1 = 0.063 \cdot in^3$$

$$w_c := V_c \cdot \gamma_{st304} = 0.018 \cdot lbf$$

Tubular Cross Section

$$A_t := \frac{\pi \cdot \left(D_o^{\,2} - d_i^{\,2}\right)}{4} = 0.02 \cdot in^2$$

$$V_t := A_t \cdot L_3 = 0.09 \cdot in^3$$

$$w_t := V_t \cdot \gamma_{st304} = 0.026 \cdot lbf$$

Where A is the cross sectional area of the torsion bar, V represents the vollume of material, and w represents the total weight of the torsion bar. Subscripts 'c' and 't' have been assigned to represent the values calculated for a closed circular cross section and a tubular cross section respectively.

Appendix D: Torque Required to Lift Dead Weight of Leg

GIVEN: A stroke victom suffering from hemiperises, has trouble completing a bycicle crank cycle, specificly between the angle of 180 degrees and 245 degrees, where 0 degrees is when the crank is in line with the positive Y-Axis (refer to figure 1 for this cordinate system).

FIND: The required assistive torque to aid the victom in completing the crank cycle between the angles of 180 degrees and 245 degrees.

ASSUMPTIONS: 1.) 10% of the assistive torque will be lost due to friction.
2.) The dead weight of the leg is 17% of the users weight
3.) The assumed weight of the user is 160 lbf
4.) The F_{leg} will be considerd as the dead weight of the leg

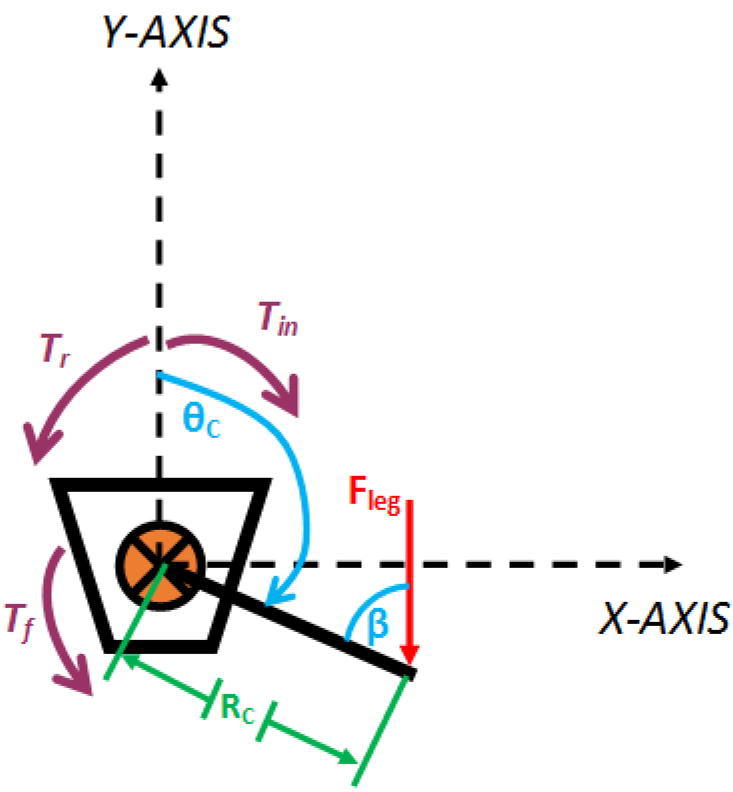


Figure 1: Free Body Diagram of the Crank

Where:

θ_C = The angle of the crank
β = The angle between the crank and the downward acting force of the leg
R_C = The Length of the crank
F_{leg} = The Downward force of the leg
T_{in} = The input torque, required to move the bike, generated during the first 180 degrees of the crank cycle.
T_r = The reaction torque, equal and opposite to the input torque, is generated during the last 180 degrees of the crank cycle.
T_f = The resistance torque caused by friction within the system

KNOWN:

$w_{person} := 160lbf$ $w_{leg} := .17 \cdot w_{person} = 27.2 \cdot lbf$ $R_C := 8in$

$F_{leg} := w_{leg} = 27.2 lbf$ $\theta_c := 0deg, 1deg .. 360deg$

CALCULATIONS:

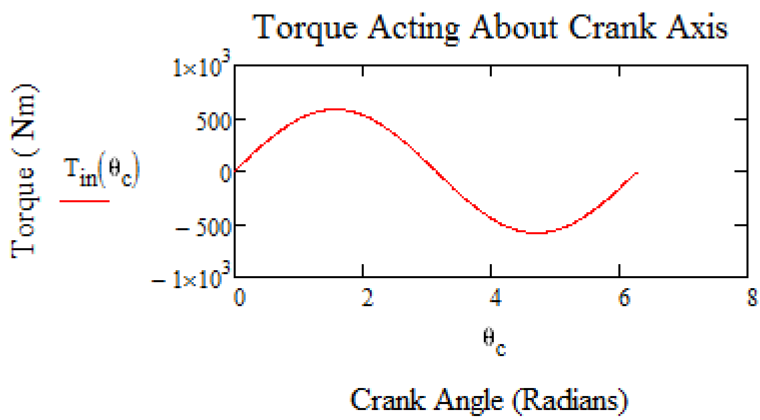
$\beta(\theta_c) := 180deg - \theta_c$

The input and reaction torque can be calculated by multiplying the the perpendicular component of the leg force by the length of the crank. The perpendicular component of the leg force will be represented as F_p and can be calculated as:

$$F_p(\theta_c) := F_{leg} \cdot \cos(90deg - \beta(\theta_c))$$

The torque caused by the weight of the leg can then be calculated for any crank angle θ_c as:

$$T_{in}(\theta_c) := F_p(\theta_c) \cdot R_C$$



Plot 1: The Torque caused by the weight of the leg through one rotation of the crank

The maximum torques will be caused when the force of the leg acts perpendicularly to the crank, thus the maximum torques will be:

$T_{max} := T_{in}(90deg)$ $T_{min} := T_{in}(270deg)$

$T_{max} = 217.6 \cdot lbf \cdot in$ $T_{min} = -217.6 \cdot lbf \cdot in$

The above torques were calculated without considering the friction assumption, takeing assumption 1 into account we have:

$T_f := .1 \cdot T_{max} = 21.76 \cdot lbf \cdot in$

where T_f in this case is the maximum frictional torque experienced through the crankcycle, to design a device that will assist the user in lifting the dead weight of their leg the device will need to supply a torque capable of overcoming this frictional torque, thus we have:

$T_{actual} := T_{max} + T_f = 239.36 \cdot lbf \cdot in$

Since the constraints of the problem state that the range in which aid needs to be supplied is between 180 and 245 degrees, these maximum torque values lie outside of the desired crank angle range. We will now consider the crank angle range we are attempting to aid the user with, the maximum torque requried in our range will occure at 245 degrees, thus we have:

$T_{rangeMAX} := T_{in}(245deg)$

$T_{rangeMAX} = -197.213 \cdot lbf \cdot in$

$T_{fRANGE} := .1 \cdot T_{rangeMAX} = -19.721 \cdot lbf \cdot in$

$T_{actualRANGE} := T_{rangeMAX} + T_{fRANGE} = -216.934 \cdot lbf \cdot in$

That is that the maximum torque our device will need to supply to the crank will be approximatly 217 lbf*in. Through supplying this torque to the crank, the bike will be able to maintain constant velocity, during the section of pedal stroke that the user is unable to supply a force.

Appendix E: Pulley Design Analysis

GIVEN: The mechanism depicted in figure 1, a pully diameter of 2.5 inches, and a desired torque at the crank of 217 lbf*in

FIND: The force required from the spring to provide the desired torque about the crank

ASSUMPTIONS: 1.) 10% of the spring force will be lost due to friction

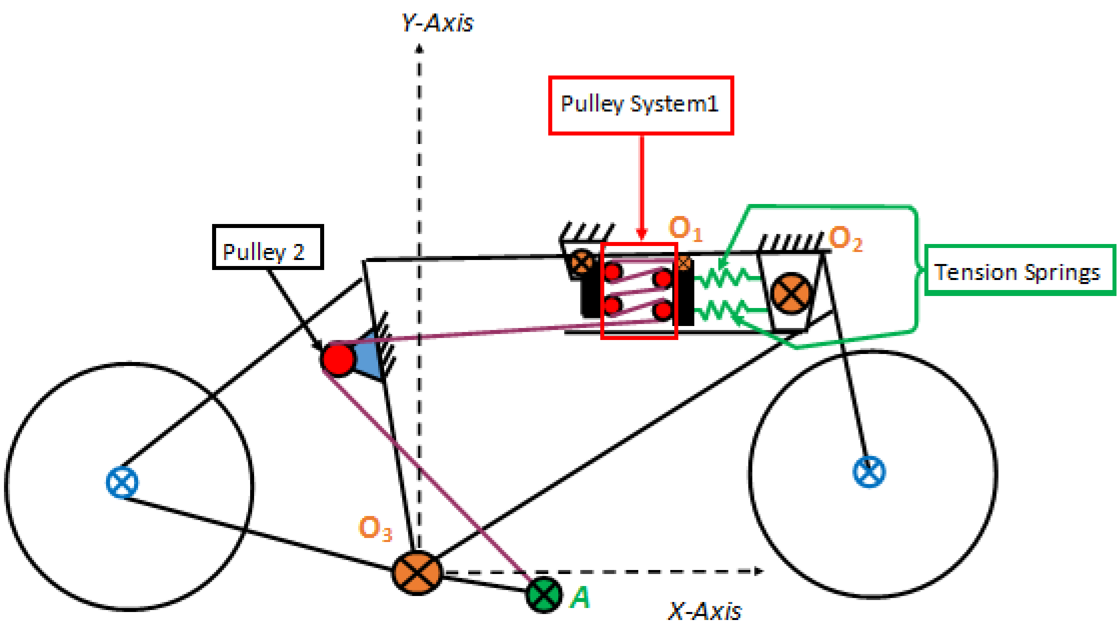


Figure 1: Depiction of Mechanism on Bike

Where:

-The Purple Lines represent a continuous cable grounded at O_1, and connected to the bike pedal at point A.

-The green tension springs provide the force in the system, and is grounded at O_2

-Pulley System 1 serves to decrease the linear displacement needed by the tension spring to obtain the necisary 16 inches cable displacement

-Pulley 2 represents a guidance pulley with a diameter of 2.5 inches used to redirect the cable to the pedal

-Point A represents the point at which the cable attaches to the pedal

-O_3 represents the axis of rotation for the bycicle crank, and origin of the global position axis

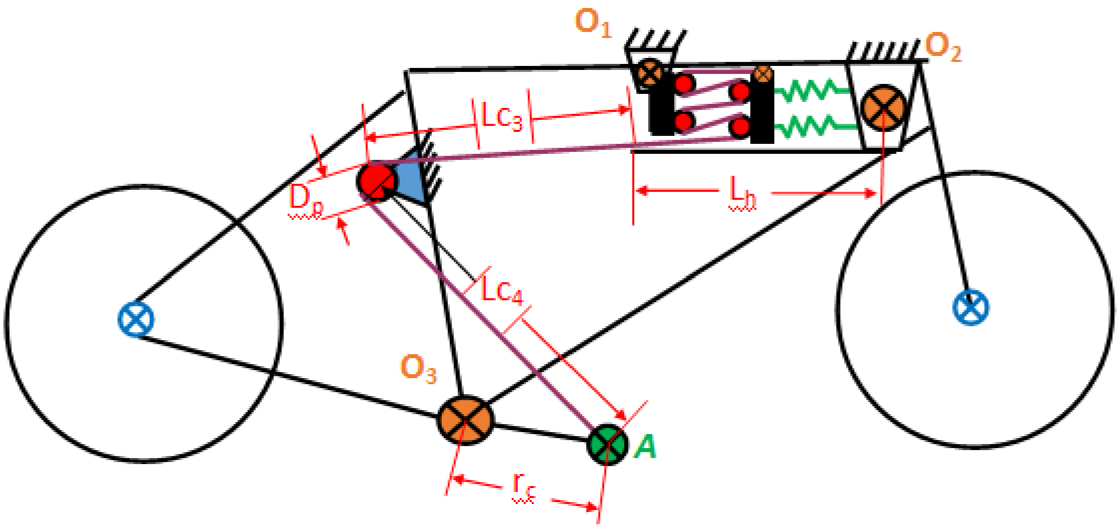


Figure 2: Length variables of mechanism

Where:

r_c = The length of the pedal crank

Lc_3 = The length of cable measured from the center of roller 1 to the center of the guidance pulley (pulley 2)

Lc_4 = The length of cable from the center of the guidance pulley (pulley 2) to the attachment point on the pedal (point A)

L_h = The length of the housing for the spring pulley mechanism, from where the spring is grounded at O_2 to the center of roller 1.

D_{pully} = The diameter of the guidence pully (pulley 2)

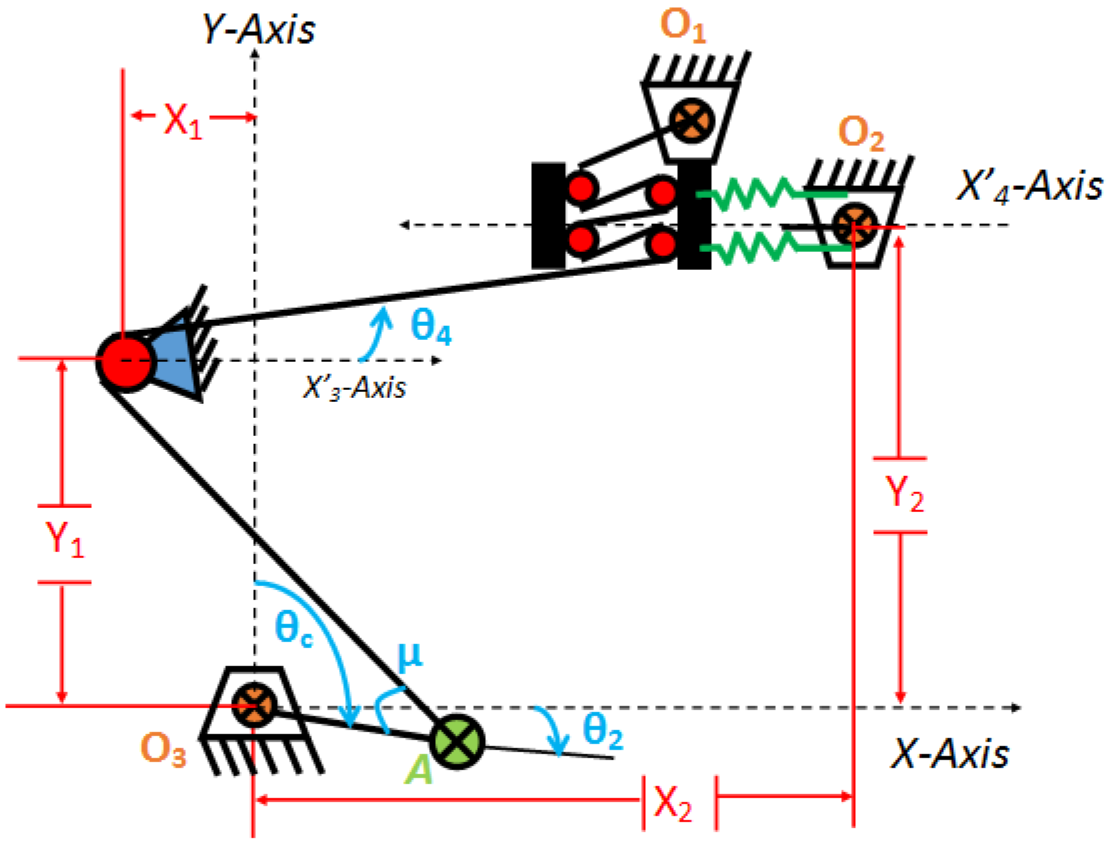


Figure 3: Isolated Mechanism, with Relevent Angles

Where:

θ_c = The angle of the pedal crank
θ_2 = The angle of link 2 (r_c) from the x-axis
θ_4 = The angle of link 5 from the x'_3-axis
μ = The transmission angle between link 2 and link 3
Y_1= The vertical offset from O_2 to the center of the guidence pully
X_1 = The horizontal offset of the guidence pully
Y_2 = The vertical offset of O_4 from O_2
X_2 = The horizontal offset of O_4 from O_2
O_3 = The center of rotation of the pedal crank
O_2= The point at which the spring is grounded
O_1 = The point at which the cable is grounded

POSITION ANALYSIS

KNOWN VARIABLES:

$r_c := 8in$ $\quad X_1 := 4in$ $\quad Y_1 := 20in$ $\quad X_2 := 24in$ $\quad Y_2 := 22in$

$x_h := 8in$ $\quad D_{pully} := 2.5in$

$$r_p := \frac{D_{pully}}{2}$$

$$\theta c := 0..360$$

$$\theta_{\theta c} := \frac{2\pi}{360}\theta c$$

CALCULATIONS:

To begin we will create a function to find θ_2 for a given θ_c:

$$\theta_2(\theta_c) := \theta_c - 90deg$$

Now we will calculate θ_4, since Lc_3 will remain the same length this can be calculated using the pythagorean therom, in conjuntion with a sine or cosine function.

$$L_5 := \sqrt{(X_1 + X_2 - x_h)^2 + \left[Y_2 - (Y_1 + r_p)\right]^2}$$

$$\theta_4 := \mathrm{asin}\left[\frac{\left[Y_2 - (Y_1 + r_p)\right]}{L_5}\right] = 2.148 \cdot deg$$

and now solving for the x and y cordinates of joint A we have:

$$A_x(\theta_c) := r_c \cdot \cos(\theta_2(\theta_c)) \qquad A_y(\theta_c) := r_c \cdot \sin(\theta_2(\theta_c))$$

In order to calulate the required force of the spring we will need to know the transmission angle μ, to find this we can use the law of cosines, refer to figure 4 below for a depiction of the triangle we will use for this calculation, it is importnat to note that side C of this triangle is not part of the mechanism itself, but rather a distance between O_2 and and the guidance pulley.

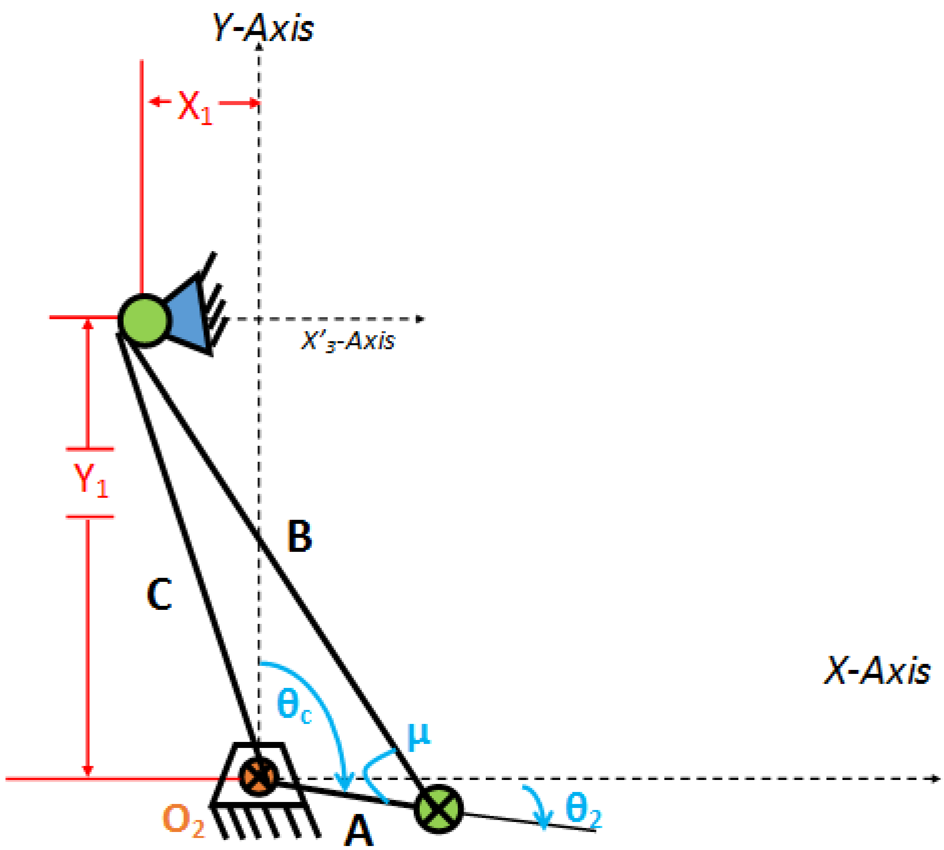


Figure: 4: The triangle for use with the law of cosines

Side C can be approximated as:

$$C := \sqrt{(X_1 + r_p)^2 + Y_1^2}$$

for any angle θ_c the side length B can be approximated as:

$$B(\theta_c) := \sqrt{(X_1 + r_p + A_x(\theta_c))^2 + (Y_1 + A_y(\theta_c))^2}$$

Side A is the crank and thus has a length of $r_c = 8$ in

$$A := r_c = 8 \cdot in$$

and now using the law of cosines we can calculate the angle μ for any given crank angle θ_c

$$\mu(\theta_c) := \mathrm{acos}\left(\frac{A^2 + B(\theta_c)^2 - C^2}{2 \cdot A \cdot B(\theta_c)}\right)$$

FORCE CALCULATIONS:

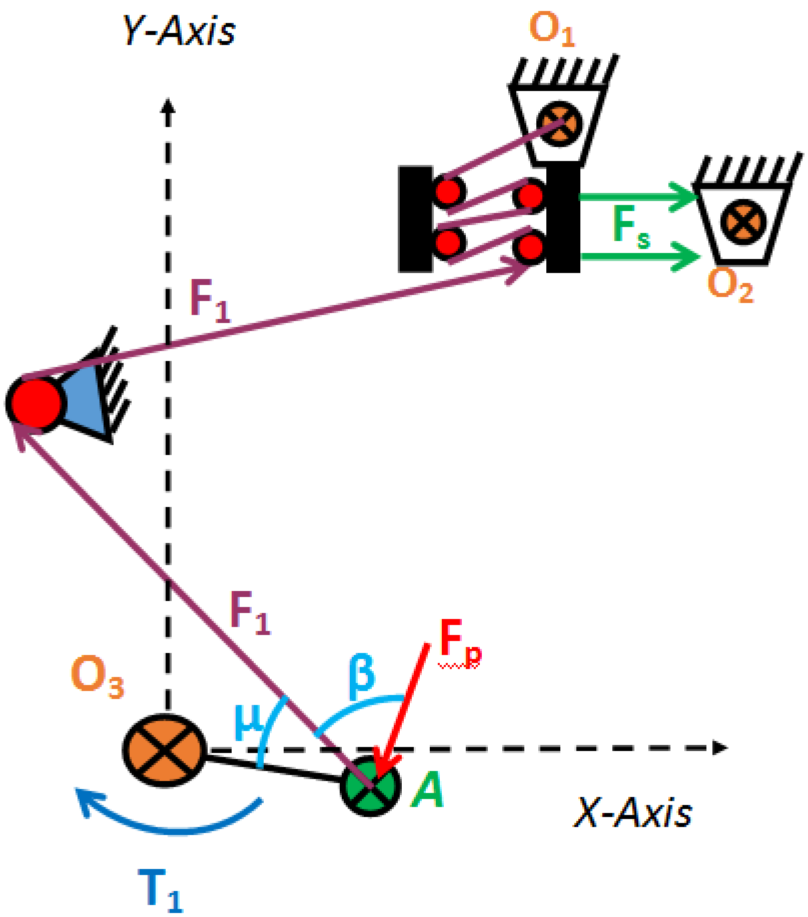


Figure 4: Representation of relevent varibles for force calculations

Where:

F_s = The total force of the spring system

F_1 = The Force acting through the cable due to the spring force

F_p = The perpendicular force component of F_1, acting perpendicular to the crank at point A

T_1 = The required torque to rotate the crank with the dead weight of the users leg through the desired aid angle

β = the angle between the applied force F_1 and its perpendicular component F_p

μ = the transmission angle between the crank and the attached cable

O_1 = The axis of rotation for the crank

KNOWN:

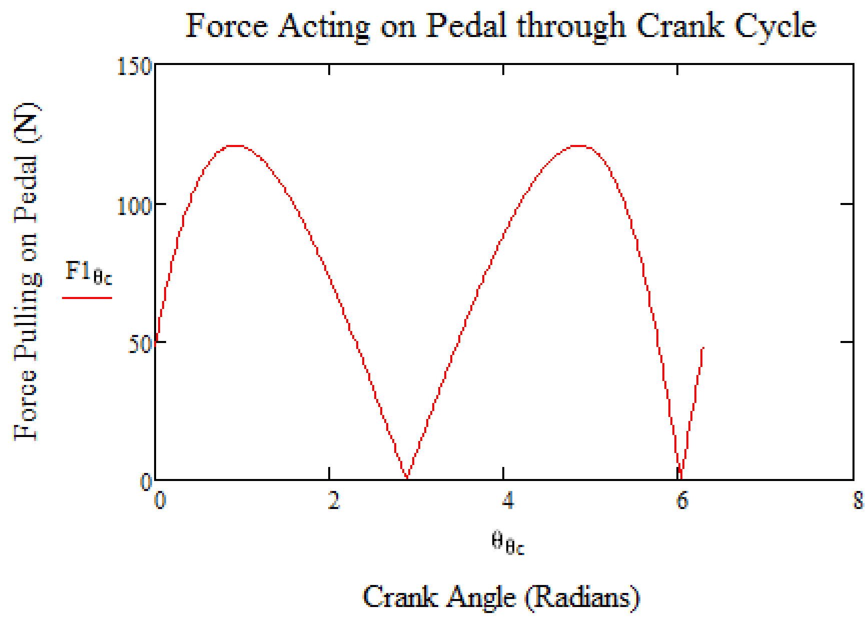
$T_1 := 217\,\text{lbf}\cdot\text{in}$ $\qquad r_c = 8\cdot\text{in}$

To calculate the required force F_1 we must first devolope a function for the angle β for any crank angle θ_c, thus we have:

$$\beta(\theta_c) := 90\text{deg} - \mu(\theta_c)$$

Now using the required torque that was calculated in previous analysis, we can determine the force F_1 required to act through the cable.

$$F1_{\theta c} := \frac{T_1}{r_c}\cdot\cos\left(\beta\left(\theta_{\theta c}\right)\right)$$



Plot 1: Force Acting on Pedal Through Crank Cycle

$F_{1MAX} := \max(F1) = 27.125\cdot\text{lbf}$

$\theta_{cMaxF} := \text{match}(\max(F1), F1) = (278)$

if we aproximate a 10% force loss due to friction, and consider that pulley 1 depicted in figure 1 will divide the supplied spring force equally into the four cable sections we can determine the necisary spring force as:

$F_{friction} := .1\cdot F_{1MAX} = 2.712\cdot\text{lbf}$

$F_{spring} := 4\cdot\left(F_{friction} + F_{1MAX}\right) = 119.35\cdot\text{lbf}$

Appendix F: Interpolations for Determining Stress Concentration Factors

DETERMINING THE A VALUE FOR THE TORSIONAL GEOMETRIC STRESS CONCENTRATION FACTOR:

$$AT := \begin{pmatrix} .86331 \\ .84897 \\ .83425 \\ .90337 \end{pmatrix} \qquad D := \begin{pmatrix} 2^3 & 2^2 & 2 & 1 \\ 1.33^3 & 1.33^2 & 1.33 & 1 \\ 1.20^3 & 1.20^2 & 1.20 & 1 \\ 1.09^3 & 1.09^2 & 1.09 & 1 \end{pmatrix}$$

$$D11 := D^T \cdot D = \begin{pmatrix} 74.198 & 40.189 & 22.614 & 13.376 \\ 40.189 & 22.614 & 13.376 & 8.397 \\ 22.614 & 13.376 & 8.397 & 5.62 \\ 13.376 & 8.397 & 5.62 & 4 \end{pmatrix}$$

$$A11 := D^T \cdot AT$$

$$Ut := D11^{-1} \cdot A11 \qquad Ut = \begin{pmatrix} -3.522 \\ 15.839 \\ -23.037 \\ 11.756 \end{pmatrix}$$

this is the polynomial we will use to calculate the 'A' value needed:

$$At(x) := Ut_0 \cdot x^3 + Ut_1 \cdot x^2 + Ut_2 \cdot x + Ut_3$$

$$A_t := At(6.18869) = -358.934$$

where:

$Ut_0 = -3.522$

$Ut_1 = 15.839$

$Ut_2 = -23.037$

$Ut_3 = 11.756$

DETERMINING THE 'b' VALUE FOR THE TORSIONAL GEOMETRIC STRESS CONCENTRATION FACTOR:

$$B := \begin{pmatrix} -.23865 \\ -.23161 \\ -.21649 \\ -.12692 \end{pmatrix} \qquad D := \begin{pmatrix} 2^3 & 2^2 & 2 & 1 \\ 1.33^3 & 1.33^2 & 1.33 & 1 \\ 1.20^3 & 1.20^2 & 1.20 & 1 \\ 1.09^3 & 1.09^2 & 1.09 & 1 \end{pmatrix}$$

$$D1 := D^T \cdot D$$

$$B1 := D^T \cdot B$$

$$Utb := D1^{-1} \cdot B1 \qquad Utb = \begin{pmatrix} -3.05 \\ 13.951 \\ -20.755 \\ 9.871 \end{pmatrix}$$

this is the polynomial we will use to calculate the b value for any D/d ratio:

$$bt(x) := Utb_0 \cdot x^3 + Utb_1 \cdot x^2 + Utb_2 \cdot x + Utb_3$$

where:

$Utb_0 = -3.05$

$Utb_1 = 13.951$

$Utb_2 = -20.755$

$Utb_3 = 9.871$

now calculating the neuber constant to find the notch sensitivity q: (for steels, Data from PG. 378 of Machine Design Text)

$$Sut := \begin{pmatrix} 50 \\ 55 \\ 60 \\ 70 \\ 80 \\ 90 \\ 100 \\ 110 \\ 120 \\ 130 \\ 140 \\ 160 \\ 180 \\ 200 \\ 220 \\ 240 \end{pmatrix} \qquad Neub := \begin{pmatrix} 0.130 \\ 0.118 \\ 0.108 \\ 0.093 \\ 0.080 \\ 0.070 \\ 0.062 \\ 0.055 \\ 0.049 \\ 0.044 \\ 0.039 \\ 0.031 \\ 0.024 \\ 0.018 \\ 0.013 \\ 0.009 \end{pmatrix} \qquad guess := \begin{pmatrix} 0.01 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$Neub = f(Sut, C3, C2, C1, C0)$$

$$f(x, C3, C2, C1, C0) := C3 \cdot x^3 + C2 \cdot x^2 + C1 \cdot x + C0$$

$C3 := guess_0$

$C2 := guess_1$

$C1 := guess_2$

$C0 := guess_3$

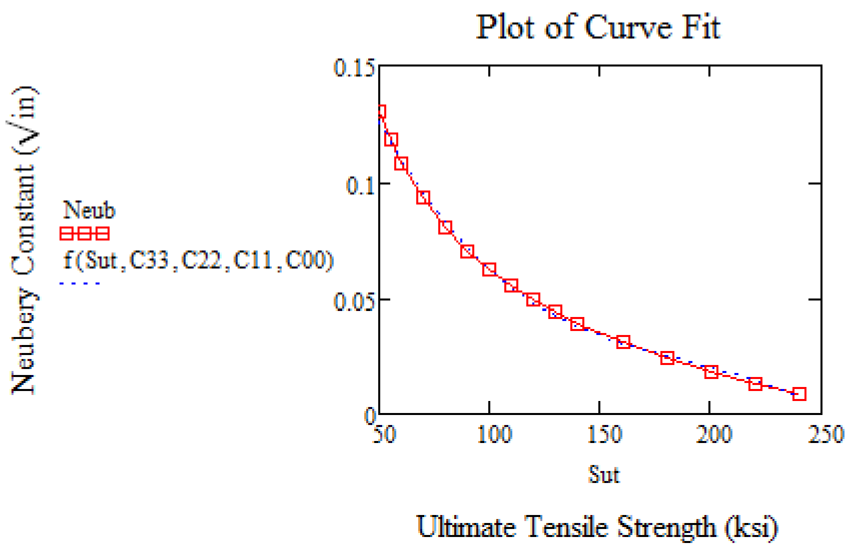
$$sol := genfit(Sut, Neub, guess, f) = \begin{pmatrix} -2.67 \times 10^{-8} \\ 1.509 \times 10^{-5} \\ -3.078 \times 10^{-3} \\ 0.246 \end{pmatrix}$$

$C33 := sol_0$

$C22 := sol_1$

$C11 := sol_2$

$C00 := sol_3$



this evaluates the function for a ultimate tensile strength of 185 ksi

$neub1 := f(185, C33, C22, C11, C00) = 0.024$

now calculating the neuber constant to find the notch sensitivity q: (for Hardened Aluminum Data from PG. 378 of Machine Design Text)

$$Sut := \begin{pmatrix} 15 \\ 20 \\ 30 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \\ 90 \end{pmatrix} \qquad Neub := \begin{pmatrix} 0.475 \\ 0.380 \\ 0.278 \\ 0.219 \\ 0.186 \\ 0.162 \\ 0.144 \\ 0.131 \\ 0.122 \end{pmatrix} \qquad guess := \begin{pmatrix} 0.01 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$Neub = f(Sut, C3, C2, C1, C0)$$

$$f(x, C3, C2, C1, C0) := C3 \cdot x^3 + C2 \cdot x^2 + C1 \cdot x + C0$$

$C3 := guess_0$

$C2 := guess_1$

$C1 := guess_2$

$C0 := guess_3$

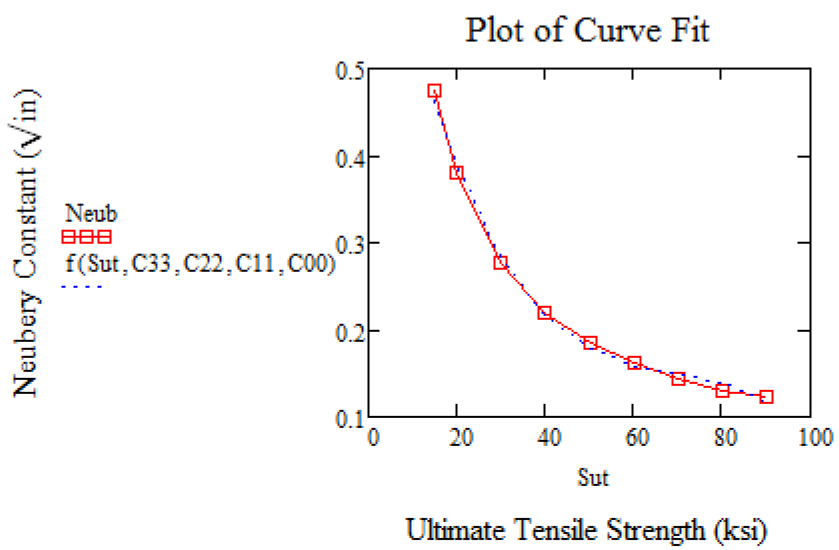
$$sol := genfit(Sut, Neub, guess, f) = \begin{pmatrix} -1.69 \times 10^{-6} \\ 3.508 \times 10^{-4} \\ -0.025 \\ 0.768 \end{pmatrix}$$

$C33 := sol_0$

$C22 := sol_1$

$C11 := sol_2$

$C00 := sol_3$



this evaluates the function for a ultimate tensile strength of 185 ksi

$neub2 := f(83, C33, C22, C11, C00) = 0.134$

now calculating the neuber constant to find the notch sensitivity q: (for Annealed Aluminum Data from PG. 378 of Machine Design Text)

$$Sut := \begin{pmatrix} 10 \\ 15 \\ 20 \\ 25 \\ 30 \\ 35 \\ 40 \\ 45 \end{pmatrix} \qquad Neub := \begin{pmatrix} 0.500 \\ 0.341 \\ 0.264 \\ 0.217 \\ 0.180 \\ 0.152 \\ 0.126 \\ 0.111 \end{pmatrix} \qquad guess := \begin{pmatrix} 0.01 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$Neub = f(Sut, C3, C2, C1, C0)$$

$$f(x, C3, C2, C1, C0) := C3 \cdot x^3 + C2 \cdot x^2 + C1 \cdot x + C0$$

$C3 := guess_0$

$C2 := guess_1$

$C1 := guess_2$

$C0 := guess_3$

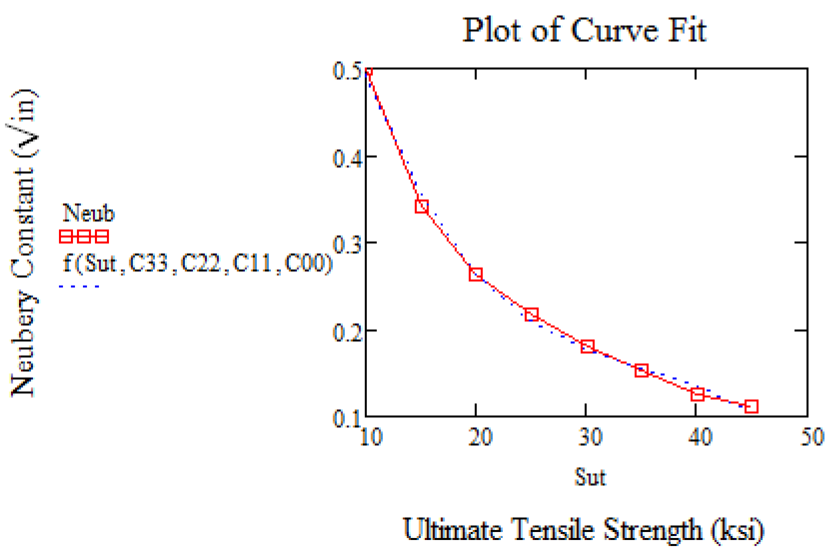
$$sol := genfit(Sut, Neub, guess, f) = \begin{pmatrix} -1.521 \times 10^{-5} \\ 1.615 \times 10^{-3} \\ -0.061 \\ 0.954 \end{pmatrix}$$

$C33 := sol_0$

$C22 := sol_1$

$C11 := sol_2$

$C00 := sol_3$



this evaluates the function for a ultimate tensile strength of 185 ksi

$neub3 := f(43, C33, C22, C11, C00) = 0.12$

Appendix G: Guidance Pulley Mounting Plate Tear-Out Analysis

Guidance Pulley Mounting Plate, TEAR-OUT ANALYSIS:

GIVEN: The Provided mounting plate design, and the forces induced on the guidance pulley in the pulley mechanism design.

FIND: 1.) The minimum wall thickness for mounting holes to design against failure due to tear-out
2.) The weight of the plate

ASSUMPTIONS: 1.) The material of the plate is acrylic

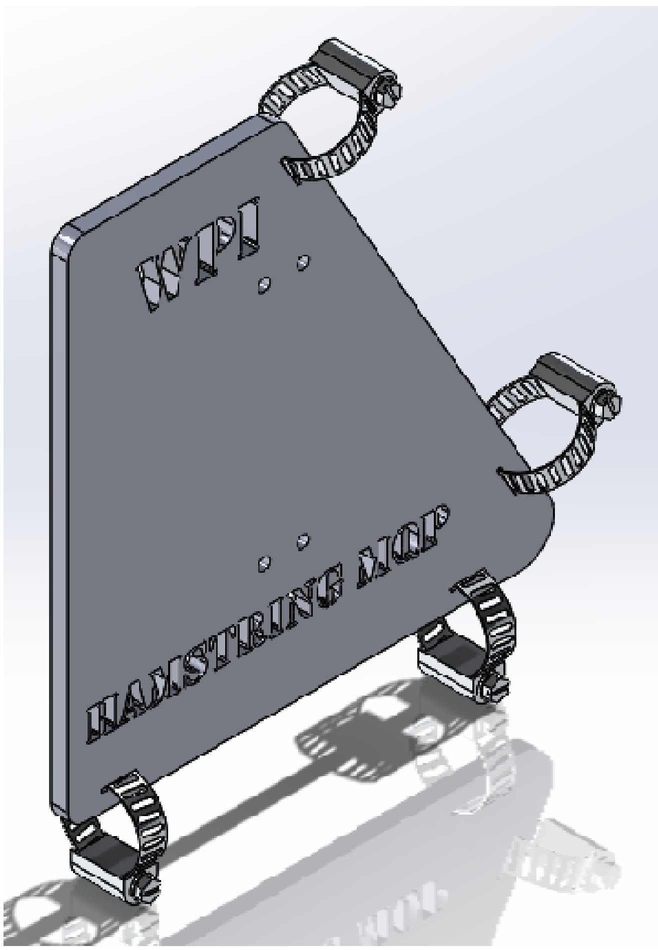


Figure 1: Part Model of Mounting Plate with Aircraft Clamps in Place

Where: The aircraft clamps will be clamped to the rear triangle of the bycicle, and the guidance pulley will be mounted in the four mounting holes.

Before we can begin our tearout analysis we must first determin the forces that the mounting bracket will be subjected to, refer to figure 2 below for a visual representation of these forces.

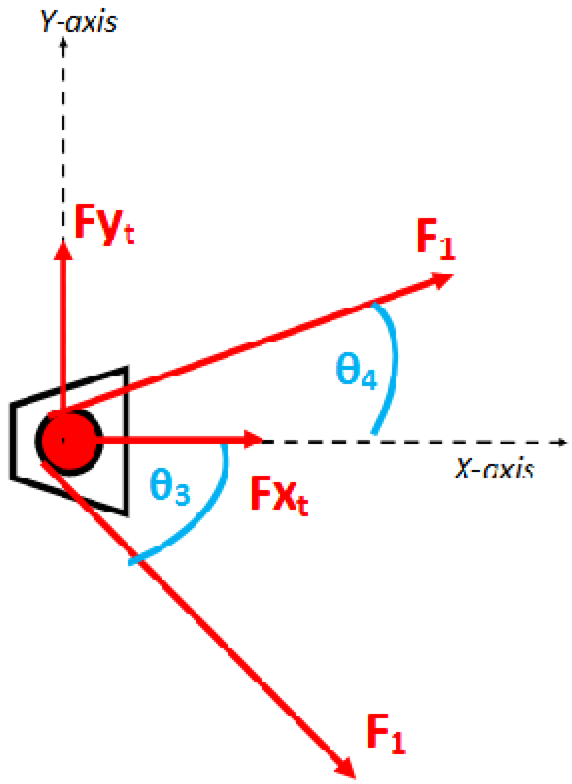


Figure 2: Forces acting on pulley

Where:

F_1 = the tension force acting through the cable as a result of the spring force

Fx_t = the total force acting in the x direction

Fy_t = the total force acting in the y direction

θ_3 = the angle between the x-axis and the force through the cable between the pulley and the crank

θ_4 = the angle between the x-axis and the cable running from the guidance pulley to the spring housing

KNOWN VARIABLES:

$F_1 := 27.5\text{lbf}$

$\theta_4 := 2.148\text{deg}$

CALCULATIONS:

X and Y components of Forces

$$Fx_4 := F_1 \cdot \cos(\theta_4) = 27.481 \cdot lbf$$

$$Fy_4 := F_1 \cdot \sin(\theta_4) = 1.031 \cdot lbf$$

To calculate θ_3 we need to determine the position of the crank for the instance in which the most force will be applied through the cable, this will occur at the crank angle at which the crank is inline with the cable, at this point the spring will be at its maximum extension, and thus exerting the maximum amount of force through the cable.

$X_1 := 4in$ $\quad$ $Y_1 := 20in$ $\quad$ $D_p := 2.117in$

$$r_p := \frac{D_p}{2}$$

$Adj := X_1 + r_p$ $\quad$ $opp := Y_1$

$$\theta_3 := atan\left(\frac{opp}{Adj}\right) = 75.806 \cdot deg$$

$$Fx_3 := F_1 \cdot \cos(\theta_3) = 6.743 \cdot lbf$$

$$Fy_3 := F_1 \cdot \sin(\theta_3) = 26.66 \cdot lbf$$

since both instances of the F_1 force act in the same x-direction they can be summed to find the total x-component force acting on the pulley:

$$Fx_t := Fx_3 + Fx_4 = 34.224 \cdot lbf$$

since the two instances of the F_1 force are acting in opposite y-directions, Fy_3 must be subtracted from Fy_4.

$$Fy_t := Fy_4 - Fy_3 = -25.63 \cdot lbf$$

the negetave indicates that the total y-force will be acting in the downward direction.

These forces act at the pulley center of rotation, and can be considard as torques acting about each mounting hole. These torques will then cause forces on the aircraft clamp holes, see figure 3 below:

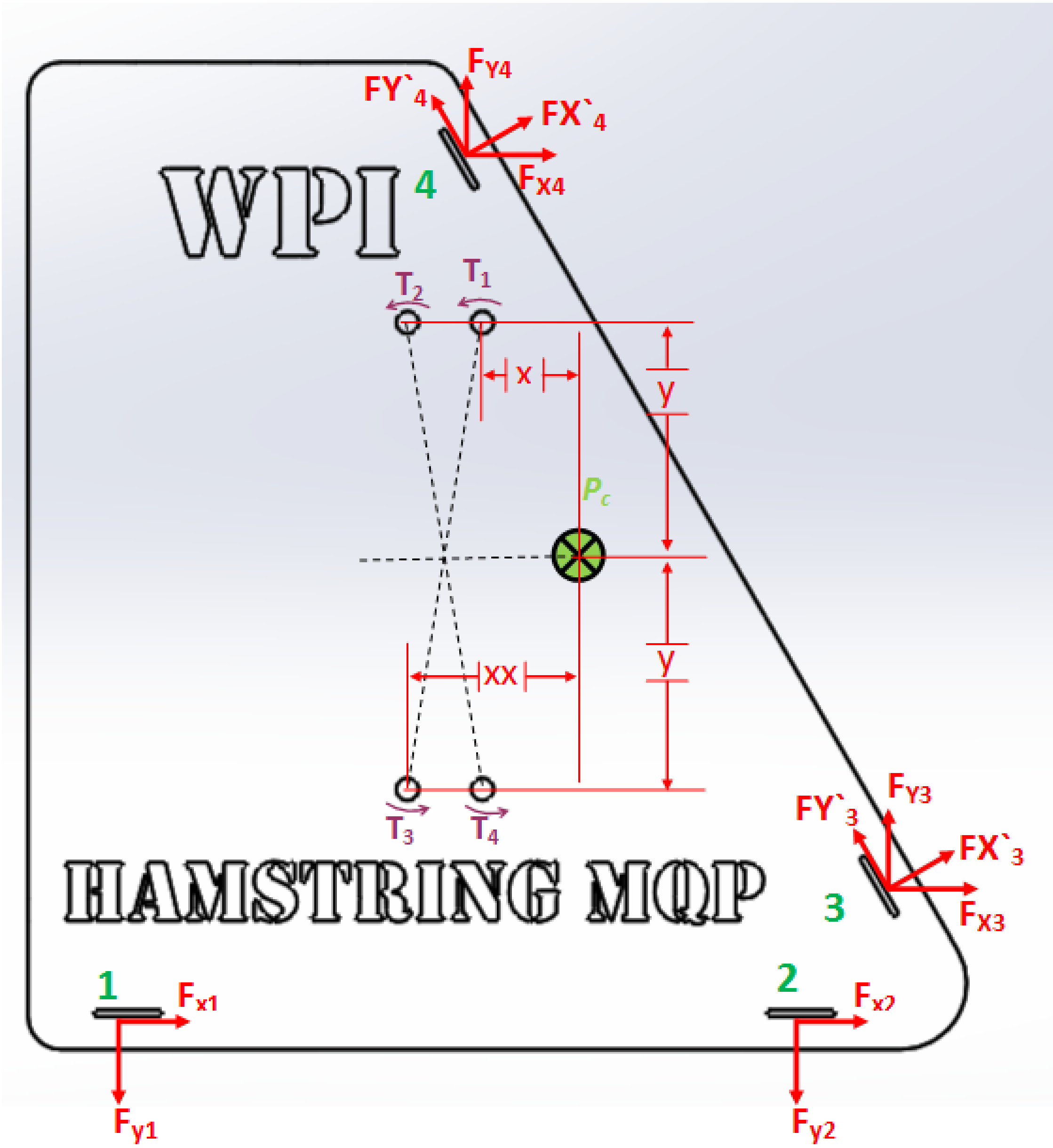


Figure 3: Representation of Forces and Pulley Geometry.

Where: P_c = The pulley center of rotation

x = The horizontal distance from P_c to pulley mounting holes 1 and 4 (represented as T_1 and T_4)

xx = The horizontal distance from P_c to pulley mounting holes 2 and 3

y = The vertical distance from P_c to the pulley mounting holes

T_1 = The torque caused about pulley mounting hole 1.

T_2 = The torque caused about pulley mounting hole 2.

T_3 = The torque caused about pulley mounting hole 3.

T_4 = The torque caused about pulley mounting hole 4.

F_{x4} = The x component of force acting at bracket mounting hole 4.

F_{y4} = The y component of force acting at bracket mounting hole 4.

$Fx`_4$ = The force acting paralell to right edge of mounting plate.

$Fy`_4$ = The force acting perpendicular to right edge of plate.

The bracket mounting holes are labled sequentially in a counter-clockwise direction. Holes 3 and 4 have similar force lables, while holes 1 and 2 only have x and y components labled. This is due to the orientation of the holes and the force components we will be considering. The force labels at each hole represent the same information displayed above for hole 4, according to the holes appropriate subscripts.

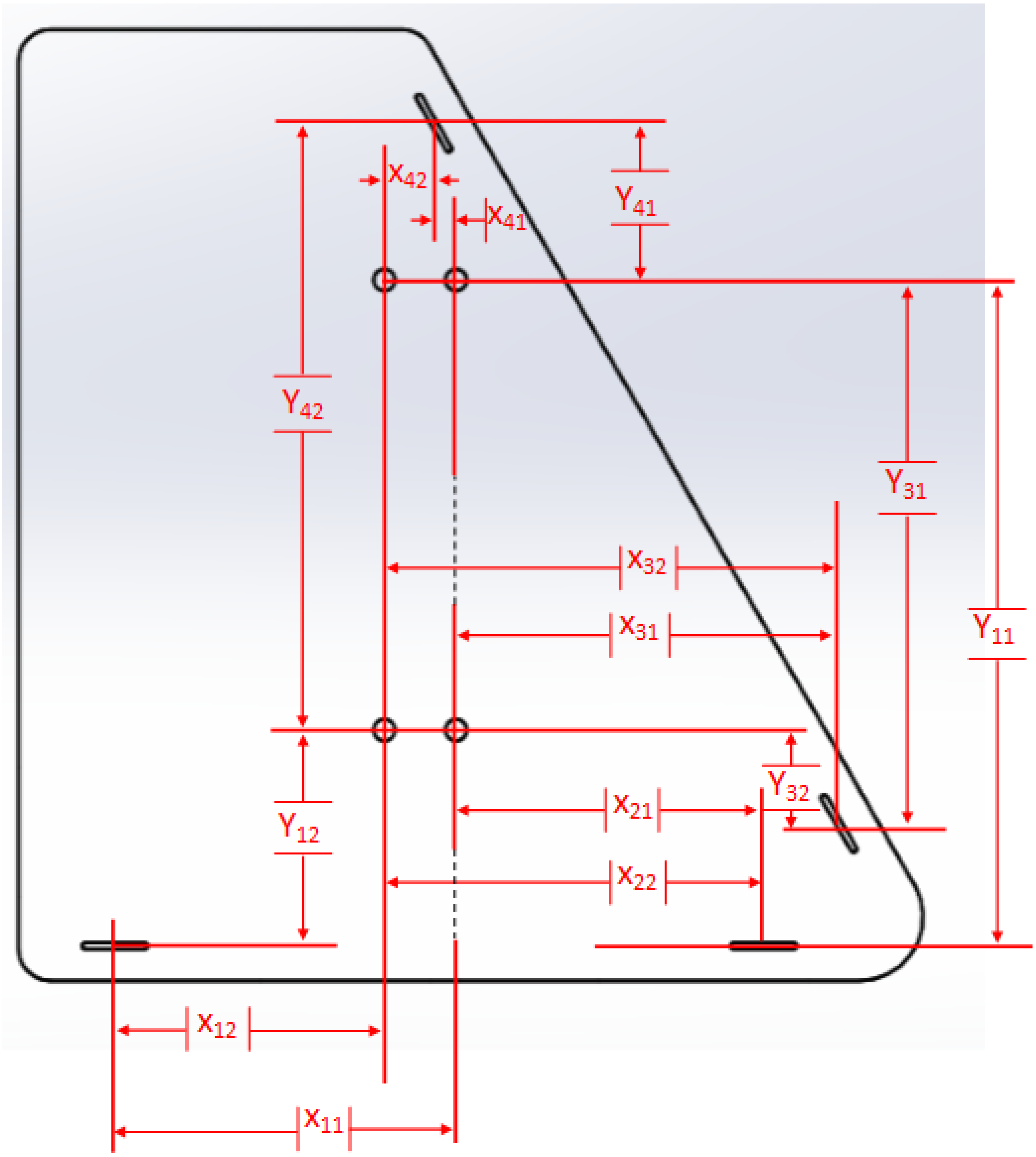


Figure 4: Length Variables for Mounting Plate.

Where: X_{11} = The horizontal distance from aircraft clamp hole 1 to the right set of pulley mounting holes.

X_{12} = The horizontal distance from aircraft clamp hole 1 to the left set of pulley mounting holes

X_{21} = The horizontal distance from the center of aircraft clamp hole 2 to the right set of pulley mounting holes.

X_{22} = The horizontal distance from the center of aircraft clamp hole 2 to the left set of pulley mounting holes

X_{31} = The horizontal distance from the center of aircraft clamp hole 3 to the right set of pulley mounting holes

X_{32} = The horizontal distance from the center of aircraft clamp hole 3 to the left set of pulley mounting holes

X_{41} = The horizontal distance from the center of aircraft clamp hole 4 to the right set of pulley mounting holes

X_{42} = The horizontal distance from the center of aircraft clamp hole 4 to the left set of pulley mounting holes

Y_{11} = The vertical distance from the center of aircraft clamp holes 1 and 2 to the center of the upper set of pulley mounting holes

Y_{12} = The vertical distance from the center of aircraft clamp hole 1 and 2 to the center of the lower set of pulley mounting holes

Y_{31} = The vertical distance from the center of aircraft clamp hole 3 to the center of the upper set of pulley mounting holes

Y_{32} = The vertical distance from the center of aircraft clamp hole 3 to the center of the lower set of pulley mounting holes

Y_{41} = The vertical distance from the center of aircraft clamp hole 4 to the center of the upper set of pulley mounting holes

Y_{42} = The vertical distance from the center of aircraft clamp hole 4 to the center of the lower set of pulley mounting holes

GIVENS FOR TEAR-OUT:

$x_{11} := 2.62in$ $\quad x_{21} := 2.35in$ $\quad x_{31} := 2.99in$ $\quad x_{41} := 0.2in$

$x_{12} := 2.06in$ $\quad x_{22} := 2.92in$ $\quad x_{32} := 3.56in$ $\quad x_{42} := 0.37in$

$y_{11} := 5.28in$ $\quad y_{31} := 4.1in$ $\quad y_{41} := 1.22in$

$y_{12} := 1.73in$ $\quad y_{32} := 0.75in$ $\quad y_{42} := 4.77in$

$y := \frac{3.55}{2} in = 0.045\,m$ $\quad xx := 1.242in - .255in = 0.987 \cdot in$

$x := xx - .57in = 0.417 \cdot in$

$Fy_t = -25.63 \cdot lbf$ $\quad Fx_t = 34.224 \cdot lbf$

TORQUE CALCULATION ON MOUNTING HOLES:

$T_{1x} := Fx_t \cdot y = 60.747 \cdot lbf \cdot in$ $\quad T_{1y} := Fy_t \cdot x = -10.688 \cdot lbf \cdot in$

$T_1 := T_{1x} + T_{1y} = 50.06 \cdot lbf \cdot in$

$T_{2x} := Fx_t \cdot y = 60.747 \cdot lbf \cdot in$ $\quad T_{2y} := Fy_t \cdot xx = -25.297 \cdot lbf \cdot in$

$T_2 := T_{2x} + T_{2y} = 35.451 \cdot lbf \cdot in$

$T_{3x} := Fx_t \cdot -y = -60.747 \cdot lbf \cdot in$ $\quad T_{3y} := Fy_t \cdot xx = -25.297 \cdot lbf \cdot in$

$T_3 := T_{3x} + T_{3y} = -86.044 \cdot lbf \cdot in$

$T_{4x} := Fx_t \cdot -y = -60.747 \cdot lbf \cdot in$ $\quad T_{4y} := Fy_t \cdot x = -10.688 \cdot lbf \cdot in$

$T_4 := T_{4x} + T_{4y} = -71.435 \cdot lbf \cdot in$

FORCES ACTING ON AIRCRAFT CLAMP MOUNTING HOLES:

FOR AIRCRAFT CLAMP 1:

$$F_{y1} := \frac{T_1}{x_{11}} + \frac{T_2}{x_{12}} + \frac{T_3}{x_{12}} + \frac{T_4}{x_{11}} = -32.718 \cdot \text{lbf}$$

$$F_{x1} := \frac{T_1}{y_{11}} + \frac{T_2}{y_{11}} + \frac{T_3}{y_{12}} + \frac{T_4}{y_{12}} = -74.833 \cdot \text{lbf}$$

FOR AIRCRAFT CLAMP 2:

$$F_{y2} := \frac{T_1}{x_{21}} + \frac{T_2}{x_{22}} + \frac{T_3}{x_{22}} + \frac{T_4}{x_{21}} = -26.422 \cdot \text{lbf}$$

$$F_{x2} := \frac{T_1}{y_{11}} + \frac{T_2}{y_{11}} + \frac{T_3}{y_{12}} + \frac{T_4}{y_{12}} = -74.833 \cdot \text{lbf}$$

FOR AIRCRAFT CLAMP 3:

$$F_{y3} := \frac{T_1}{x_{31}} + \frac{T_2}{x_{32}} + \frac{T_3}{x_{32}} + \frac{T_4}{x_{31}} = -21.36 \cdot \text{lbf}$$

$$F_{x3} := \frac{T_1}{y_{31}} + \frac{T_2}{y_{31}} + \frac{T_3}{y_{32}} + \frac{T_4}{y_{32}} = -189.115 \cdot \text{lbf}$$

$$Fy'_3 := F_{y3} \cdot \sin(60\text{deg}) + F_{x3} \cdot \cos(60\text{deg}) = -113.056 \cdot \text{lbf}$$

$$Fx'_3 := F_{y3} \cdot \cos(60\text{deg}) + F_{x3} \cdot \sin(60\text{deg}) = -174.459 \cdot \text{lbf}$$

FOR AIRCRAFT CLAMP 4:

$$F_{y4} := \frac{T_1}{x_{41}} + \frac{T_2}{x_{42}} + \frac{T_3}{x_{42}} + \frac{T_4}{x_{41}} = -243.614 \cdot \text{lbf}$$

$$F_{x4} := \frac{T_1}{y_{41}} + \frac{T_2}{y_{41}} + \frac{T_3}{y_{42}} + \frac{T_4}{y_{42}} = 37.076 \cdot \text{lbf}$$

$$Fy'_4 := F_{y4} \cdot \sin(60\text{deg}) + F_{x4} \cdot \cos(60\text{deg}) = -192.438 \cdot \text{lbf}$$

$$Fx'_4 := F_{y4} \cdot \cos(60\text{deg}) + F_{x4} \cdot \sin(60\text{deg}) = -89.698 \cdot \text{lbf}$$

TEAR OUT CALCULATIONS:

Using equation 4.9 from pg 185 of Robert L. Norton's Machine Design Text we have the shear stress equation to be used in a tearout analysis, equation 4.9 is reproduced below.

$$\tau_{xy} := \frac{P}{A_{shear}}$$

Where P is the load applied to the hole being analized for tearout, and A_{shear} is the area of material being sheared, in this case it is the width of the mounting plate (w_1) multiplied by the wall thickness (t), and multiplied by 2 to account for both sides of the hole, that is

$$A_{shear} := 2 \cdot w_1 \cdot t$$

$$t_1 := .25in$$

substituting the ultimate tensile strength of the assumed material for $\tau_{xy,}$ and rearanging to solve for the minimum wall thickness we have:

For the assumed material of acrylic, from page 1026 of Robert L. Norton's Machine Design Text we have an ultimate tensile strength ,and ultimate compressive strength of:

$Sut_{acrylic} := 10ksi$ $\qquad$ $Suc_{acrylic} := 15ksi$

$$ty'_1 := \frac{\frac{|F_{y1}|}{Sut_{acrylic}}}{2 \cdot t_1} = 0.007 \cdot in \qquad ty'_2 := \frac{\frac{|F_{y2}|}{Sut_{acrylic}}}{2 \cdot t_1} = 0.005 \cdot in$$

$$tx'_1 := \frac{\frac{|F_{x1}|}{Sut_{acrylic}}}{2 \cdot t_1} = 0.015 \cdot in \qquad tx'_2 := \frac{\frac{|F_{x2}|}{Sut_{acrylic}}}{2 \cdot t_1} = 0.015 \cdot in$$

$$ty'_3 := \frac{\frac{|Fy'_3|}{Sut_{acrylic}}}{2 \cdot t_1} = 0.023 \cdot in \qquad ty'_4 := \frac{\frac{|Fy'_4|}{Sut_{acrylic}}}{2 \cdot t_1} = 0.038 \cdot in$$

$$tx'_3 := \frac{\frac{|Fx'_3|}{Sut_{acrylic}}}{2 \cdot t_1} = 0.035 \cdot in \qquad tx'_4 := \frac{\frac{|Fx'_4|}{Sut_{acrylic}}}{2 \cdot t_1} = 0.018 \cdot in$$

TEAR-OUT CONCLUSION:

To ensure design uniformity, we will need to choose the largest required thickness as a required minimum wall thickness for the mounting plate, thus we will choose a minimum wall thickness of 0.035 inches.

WEIGHT CALCULATION:

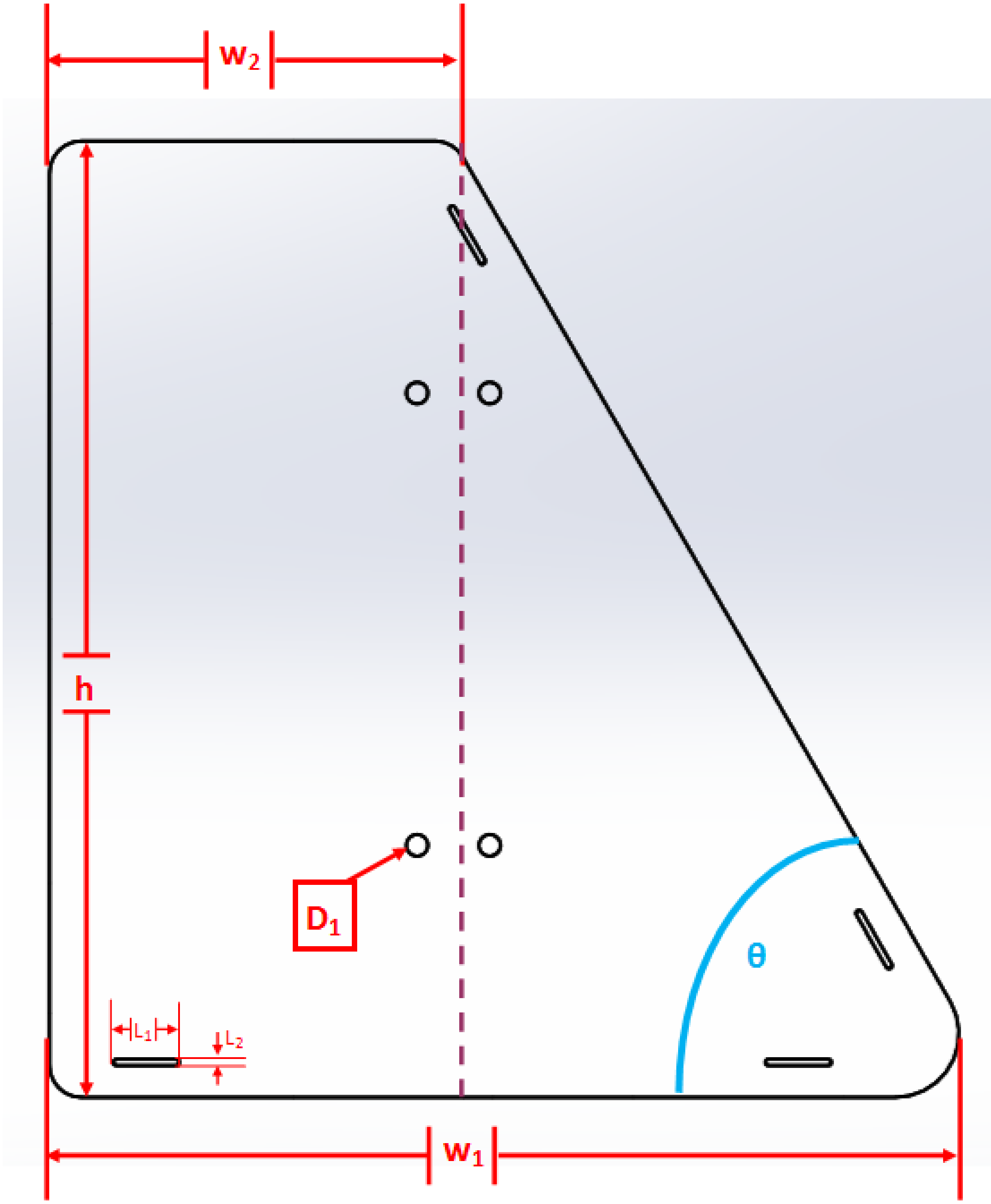


Figure 5: Dimensions of Plate, For Weight Calculation

GIVENS:

$\gamma_{acrylic} := 1.18$ — Specific gravity of acrylic, from table A-11 pg. 1026 of Robert L. Norton's Machine Design text.

$\rho_{h2o} := 1000 \frac{kg}{m^3}$ — Density of water

$\rho_{acrylic} := \rho_{h2o} \cdot \gamma_{acrylic} = 1180 \frac{kg}{m^3}$ — Density of acrylic

$w_1 := 7.5in$ — The width of the top of the plate

$h := 7.5in$ — The maximum height of the plate

$\theta := 60deg$ — Angle between bottom edge and diagonal edge

$w_2 := 3.17in$ — Width of the bottom of the plate

$w_t := w_1 - w_2 = 4.33 \cdot in$ — Width of triangle on right side of plate

$t_1 := .25in$ — Thickness of plate

$D_1 := .17in$

$L_1 := .58in$

$L_2 := .05in$

VOLLUME AND WEIGHT OF PLATE:

$V_h := \pi \cdot \left(\frac{D_1}{2}\right)^2 \cdot w_1 = 0.17 \cdot in^3$ — Volume of 1 guidance pulley mounting hole

$V_{ah} := L_1 \cdot L_2 \cdot t_1 = 7.25 \times 10^{-3} \cdot in^3$ — Volume of 1 aircraft clamp mounting hole

$V_p := \left[\left(h \cdot w_2 \cdot t_1\right) + \left(\frac{w_t \cdot h}{2} \cdot t_1\right)\right] - 4 \cdot V_{ah} - 4 \cdot V_h = 9.293 \cdot in^3$ — Total volume of plate

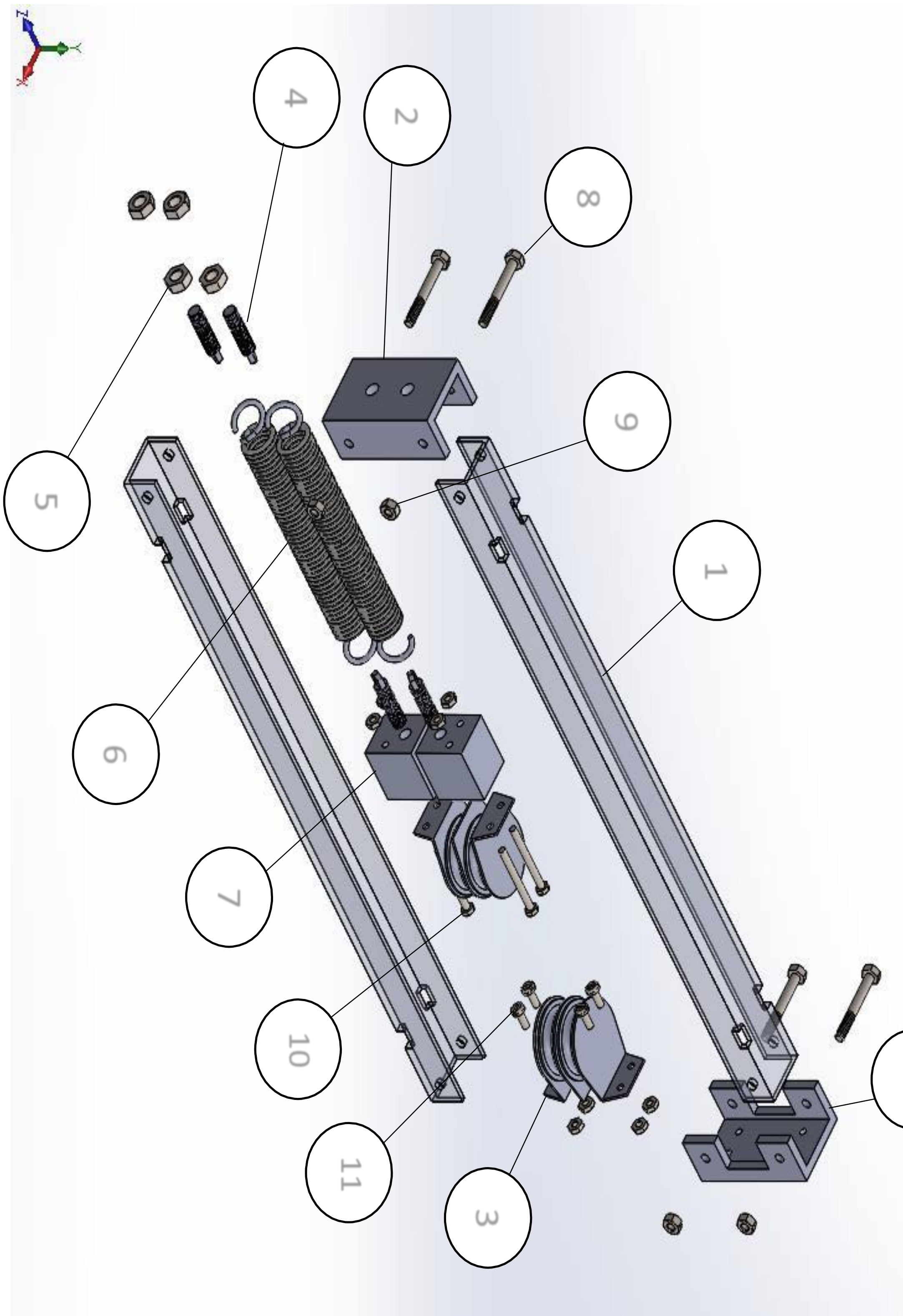
$weight_{plate} := \rho_{acrylic} \cdot g \cdot V_p = 0.396 \cdot lbf$ — Total weight of plate

Here we can see that the weight of the acryilic plate will not be a design issue

Appendix H: Decision Matrix

	Weighting	Max Points	Design	Pulley	Retractor	Quick Return	Slider
Design Requirements							
Adaptable to range of bikes		10		10	10	5	5
	3	30		30	30	15	15
Doesn't interfere with motion of bike		10		10	9	7	7
	10	100		100	90	70	70
Inrecease power to pedal (180-225)		10		8	10	10	10
	10	100		80	100	100	100
Powered soley by rider		10		10	10	10	10
	10	100		100	100	100	100
Cost less than $100		10		10	8	6	6
	4	40		40	32	24	24
Less than 5lb		10		10	8	7	7
	5	50		50	40	35	35
Maintain wieght balance of bike		10		10	8	3	3
	9	90		90	72	27	27
Smaller than 50cm by 38cm by 12cm		10		10	10	10	10
	8	80		80	80	80	80
Allow for free motion of the bike		10		10	10	10	10
	10	100		100	100	100	100
Attached and removed with basic tools		10		10	9	8	8
	5	50		50	45	40	40
Comply with ASME standard G50-10 for weather resistantce		10		10	10	10	10
	4	40		40	40	40	40
Tangential force of 138lb on the pedal from 180°-225°		10		4	10	10	10
	10	100		40	100	100	100
Maintain Proper ankle position		10		10	10	10	10
	5	50		50	50	50	50
Attatch foot to pedal		10		10	10	10	10
	10	100		100	100	100	100
Total		1030		950	979	881	881

Appendix I: Bill of Materials

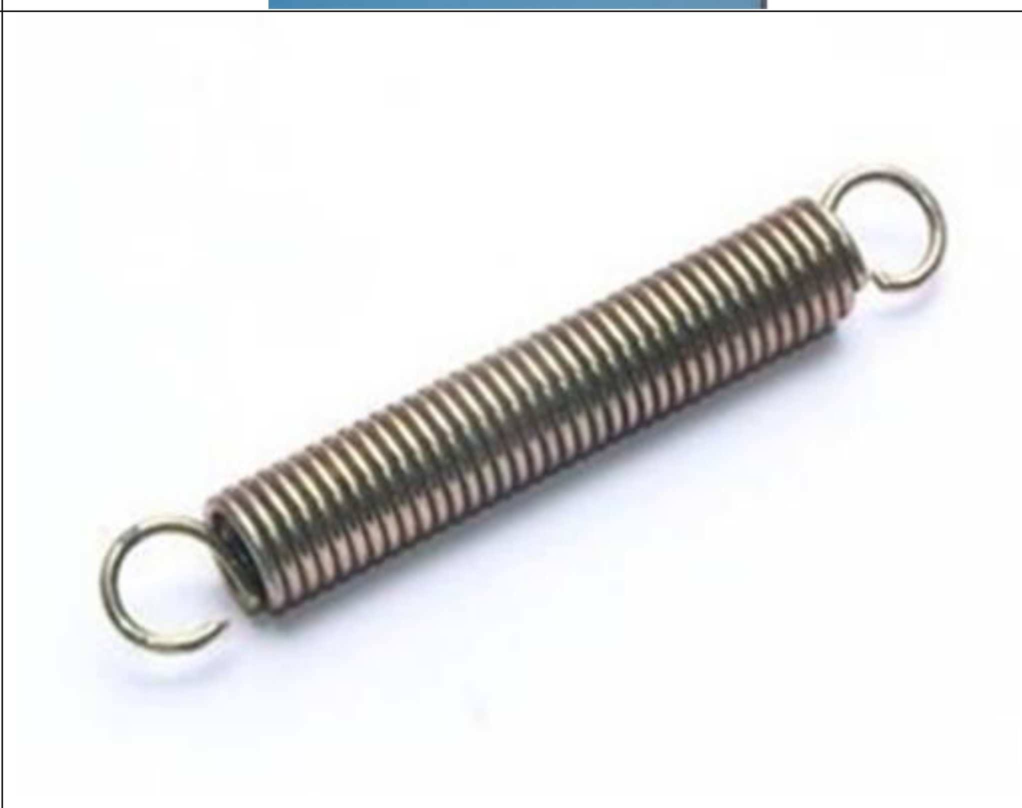


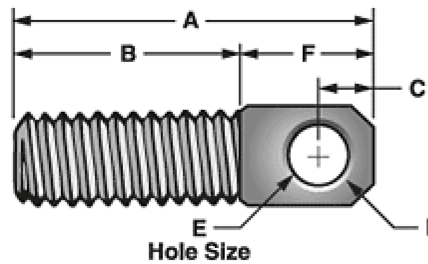
ITEM NO.	PART NUMBER	DESCRIPTION	QTY.
1	U-Channel		2
2	spring mounting bracket		1
3	Double Pully		2
4	spring anchor		4
5	machine screw nut hex_ai		12
6	Spring		2
7	Sliding Block		2
8	hex bolt_ai		4
9	hex nut_ai		4
10	hex screw_ai		4
11	pan slot head_ai		4
12	pulley mounting bracket		1

Appendix J: List of Purchased Materials

Item	Supplier	Image
Guidance pulley	MSC Industrial Supply	
Thimble Clip	MSC Industrial Supply	
Steel Wire	MSC Industrial Supply	

Double Pulley Block	Grainger Industrial Supply	
U-Bolt	MSC Industrial Supply	
U-Channel	OnlineMetals.com	

Acrylic Block	Plastic Craft	
Acrylic sheet	MSC Industrial Supply	
Extension Springs	Century Spring	

Spring Anchors	Century Spring	

Appendix K: Explanation of Spread Sheet Analysis

The following document is meant to display analysis completed on the Torsion bar design for the hamstring muscle assistance device. It is comprised of two sections, the first is numerical analysis of stress and deflection of the bar which is represented by the spread sheet analysis attached.

Stress at end of torsion bar:

The stress in a torsion bar can be calculated through the use of Equation 1, shown below:

$$S_T = \frac{16T}{\pi D^3} \quad (14)$$

Equation I 1: Stress in a Torsion Bar

The highest stress in the torsion bar will be at the ends where the design of the torsion bar will step down from a circular cross-section into a square cross section. To calculate the stress at this point we first must find an equivalent diameter for the square cross section this can be achieved through the use of equation 6.7d combined with the equation for A_{95} of a square cross section in Figure 6-25c of Robert L. Norton's Machine Design text book, on pages 363 and 364 respectively. Using this equivalent diameter a corrected endurance limit can be found using 6.6 combined with correction factors outlined in equations 6.7a-f along with table 6-4, which can be found on pages 362-367 of the machine design text.

Safety Factor for torsion bar:

The safety factor for a torsion bar can be calculated based on Equation 2 produced below:

$$n = \frac{\pi D^3}{32K_{fs}\left(\frac{T_a}{S_n} + \frac{T_m}{\sigma_e}\right)} \quad (19)$$

Equation I 2: Safety Factor for a Torsion Bar

Where D is the equivalent diameter of the torsion bar square attachment section, T_a is the alternating torque component, T_m is the mean torque component, K_{fs} is the fatigue stress concentration factor, S_n is the corrected endurance limit of the material, and σ_e is the yield strength of the material.

K_{fs} can be calculated by first calculating K_t, the geometric stress concentration factor, which can be calculated from Equation 3 below:

$$A \times \left(\frac{r}{d}\right)^b$$

Equation I 3: Stress Concentration Factor

Where r is the notch radius, d is the equivalent diameter of the square attachment end, A and b are constants which can be obtained from figure C-3 on page 1033 of Robert L. Norton's Machine Design Text book. To ensure an accurate calculation we generated a curve fit polynomial using Mathcad, using this polynomial we were able to determine the A and b values for any D/d ratio. D/d being the ratio of the Torsion bar diameter (D) to the equivalent diameter of the square cross section (d).
To find the fatigue stress concentration factor we first need to determine the notch sensitivity q, which can be equated from equation 6.13 on page 375 of Robert Norton's Machine Design text book reproduced below:

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$

Equation I 4: Notch Sensitivity

Where q is the notch sensitivity a represents the Neuber's constant which for steels can be found from table 6-6 on page 378 of Robert Norton's Machine Design text book. And r represents the notch radius.

Once we have calculated the notch sensitivity we can proceed to calculate the fatigue stress concentration factor from equation 6.11b on page 375 of Robert Norton's Machine Design text book, reproduced below:

$$K_f = 1 + q(K_t - 1)$$

Equation I 5: Fatigue Stress Concentration Factor

Where K_t is the geometric stress concentration factor as calculated previously, and q is the notch sensitivity as calculated previously.
Using this information we can calculate the safety factor using Equation 2 above.

Appendix L: Spreadsheet Analysis

Stainless Steel

Angle of Deflection

L (in)	angle of twist in degrees											
2	7373.45	460.84	91.03	28.80	11.80	5.69	3.07	1.80	1.12	0.74	0.50	0.36
3	11060.17	691.26	136.55	43.20	17.70	8.53	4.61	2.70	1.69	1.11	0.76	0.53
3.25	11981.85	748.87	147.92	46.80	19.17	9.25	4.99	2.93	1.83	1.20	0.82	0.58
3.5	12903.54	806.47	159.30	50.40	20.65	9.96	5.37	3.15	1.97	1.29	0.88	0.62
3.75	13825.22	864.08	170.68	54.00	22.12	10.67	5.76	3.38	2.11	1.38	0.94	0.67
4	14746.90	921.68	182.06	57.61	23.60	11.38	6.14	3.60	2.25	1.47	1.01	0.71
4.25	15668.58	979.29	193.44	61.21	25.07	12.09	6.53	3.83	2.39	1.57	1.07	0.76
4.5	16590.26	1036.89	204.82	64.81	26.54	12.80	6.91	4.05	2.53	1.66	1.13	0.80
4.75	17511.94	1094.50	216.20	68.41	28.02	13.51	7.29	4.28	2.67	1.75	1.20	0.84
5	18433.62	1152.10	227.58	72.01	29.49	14.22	7.68	4.50	2.81	1.84	1.26	0.89
5.25	19355.30	1209.71	238.95	75.61	30.97	14.93	8.06	4.73	2.95	1.94	1.32	0.93
5.5	20276.98	1267.31	250.33	79.21	32.44	15.65	8.45	4.95	3.09	2.03	1.38	0.98
5.75	21198.67	1324.92	261.71	82.81	33.92	16.36	8.83	5.18	3.23	2.12	1.45	1.02
6	22120.35	1382.52	273.09	86.41	35.39	17.07	9.21	5.40	3.37	2.21	1.51	1.07
6.25	23042.03	1440.13	284.47	90.01	36.87	17.78	9.60	5.63	3.51	2.30	1.57	1.11
6.5	23963.71	1497.73	295.85	93.61	38.34	18.49	9.98	5.85	3.65	2.40	1.64	1.16
6.75	24885.39	1555.34	307.23	97.21	39.82	19.20	10.36	6.08	3.79	2.49	1.70	1.20
7	25807.07	1612.94	318.61	100.81	41.29	19.91	10.75	6.30	3.93	2.58	1.76	1.24
7.25	26728.75	1670.55	329.98	104.41	42.77	20.62	11.13	6.53	4.07	2.67	1.83	1.29
7.5	27650.43	1728.15	341.36	108.01	44.24	21.34	11.52	6.75	4.21	2.77	1.89	1.33
7.75	28572.11	1785.76	352.74	111.61	45.72	22.05	11.90	6.98	4.35	2.86	1.95	1.38
8	29493.80	1843.36	364.12	115.21	47.19	22.76	12.28	7.20	4.50	2.95	2.01	1.42
8.25	30415.48	1900.97	375.50	118.81	48.66	23.47	12.67	7.43	4.64	3.04	2.08	1.47
8.5	31337.16	1958.57	386.88	122.41	50.14	24.18	13.05	7.65	4.78	3.13	2.14	1.51
8.75	32258.84	2016.18	398.26	126.01	51.61	24.89	13.44	7.88	4.92	3.23	2.20	1.56
9	33180.52	2073.78	409.64	129.61	53.09	25.60	13.82	8.10	5.06	3.32	2.27	1.60
9.25	34102.20	2131.39	421.01	133.21	54.56	26.31	14.20	8.33	5.20	3.41	2.33	1.64
9.5	35023.88	2188.99	432.39	136.81	56.04	27.02	14.59	8.55	5.34	3.50	2.39	1.69
9.75	35945.56	2246.60	443.77	140.41	57.51	27.74	14.97	8.78	5.48	3.59	2.46	1.73
10	36867.24	2304.20	455.15	144.01	58.99	28.45	15.35	9.00	5.62	3.69	2.52	1.78
10.25	37788.93	2361.81	466.53	147.61	60.46	29.16	15.74	9.23	5.76	3.78	2.58	1.82
10.5	38710.61	2419.41	477.91	151.21	61.94	29.87	16.12	9.45	5.90	3.87	2.64	1.87
D (in)	0.0625	0.125	0.1875	0.25	0.3125	0.375	0.4375	0.5	0.5625	0.625	0.6875	0.75

0.26	0.19	0.15	0.11	0.09	0.07	0.06	0.05	0.04	0.03	0.03	0.02	0.02	0.02	0.01	0.01
0.39	0.29	0.22	0.17	0.13	0.11	0.08	0.07	0.06	0.05	0.04	0.03	0.03	0.02	0.02	0.02
0.42	0.31	0.24	0.18	0.14	0.11	0.09	0.07	0.06	0.05	0.04	0.04	0.03	0.03	0.02	0.02
0.45	0.34	0.25	0.20	0.15	0.12	0.10	0.08	0.07	0.06	0.05	0.04	0.03	0.03	0.02	0.02
0.48	0.36	0.27	0.21	0.17	0.13	0.11	0.09	0.07	0.06	0.05	0.04	0.04	0.03	0.03	0.02
0.52	0.38	0.29	0.23	0.18	0.14	0.11	0.09	0.08	0.06	0.05	0.04	0.04	0.03	0.03	0.02
0.55	0.41	0.31	0.24	0.19	0.15	0.12	0.10	0.08	0.07	0.06	0.05	0.04	0.03	0.03	0.03
0.58	0.43	0.33	0.25	0.20	0.16	0.13	0.10	0.09	0.07	0.06	0.05	0.04	0.04	0.03	0.03
0.61	0.46	0.35	0.27	0.21	0.17	0.13	0.11	0.09	0.07	0.06	0.05	0.04	0.04	0.03	0.03
0.65	0.48	0.36	0.28	0.22	0.18	0.14	0.12	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.03
0.68	0.50	0.38	0.30	0.23	0.18	0.15	0.12	0.10	0.08	0.07	0.06	0.05	0.04	0.04	0.03
0.71	0.53	0.40	0.31	0.24	0.19	0.16	0.13	0.10	0.09	0.07	0.06	0.05	0.04	0.04	0.03
0.74	0.55	0.42	0.32	0.25	0.20	0.16	0.13	0.11	0.09	0.08	0.06	0.05	0.05	0.04	0.03
0.77	0.58	0.44	0.34	0.26	0.21	0.17	0.14	0.11	0.09	0.08	0.07	0.06	0.05	0.04	0.04
0.81	0.60	0.46	0.35	0.28	0.22	0.18	0.14	0.12	0.10	0.08	0.07	0.06	0.05	0.04	0.04
0.84	0.62	0.47	0.37	0.29	0.23	0.18	0.15	0.12	0.10	0.09	0.07	0.06	0.05	0.05	0.04
0.87	0.65	0.49	0.38	0.30	0.24	0.19	0.16	0.13	0.11	0.09	0.08	0.06	0.05	0.05	0.04
0.90	0.67	0.51	0.39	0.31	0.25	0.20	0.16	0.13	0.11	0.09	0.08	0.07	0.06	0.05	0.04
0.94	0.70	0.53	0.41	0.32	0.25	0.21	0.17	0.14	0.11	0.10	0.08	0.07	0.06	0.05	0.04
0.97	0.72	0.55	0.42	0.33	0.26	0.21	0.17	0.14	0.12	0.10	0.08	0.07	0.06	0.05	0.04
1.00	0.74	0.56	0.44	0.34	0.27	0.22	0.18	0.15	0.12	0.10	0.09	0.07	0.06	0.05	0.05
1.03	0.77	0.58	0.45	0.35	0.28	0.23	0.18	0.15	0.13	0.11	0.09	0.08	0.06	0.06	0.05
1.06	0.79	0.60	0.46	0.36	0.29	0.23	0.19	0.16	0.13	0.11	0.09	0.08	0.07	0.06	0.05
1.10	0.82	0.62	0.48	0.38	0.30	0.24	0.20	0.16	0.13	0.11	0.09	0.08	0.07	0.06	0.05
1.13	0.84	0.64	0.49	0.39	0.31	0.25	0.20	0.17	0.14	0.12	0.10	0.08	0.07	0.06	0.05
1.16	0.86	0.66	0.51	0.40	0.32	0.25	0.21	0.17	0.14	0.12	0.10	0.08	0.07	0.06	0.05
1.19	0.89	0.67	0.52	0.41	0.32	0.26	0.21	0.18	0.15	0.12	0.10	0.09	0.07	0.06	0.06
1.23	0.91	0.69	0.53	0.42	0.33	0.27	0.22	0.18	0.15	0.13	0.11	0.09	0.08	0.07	0.06
1.26	0.94	0.71	0.55	0.43	0.34	0.28	0.22	0.18	0.15	0.13	0.11	0.09	0.08	0.07	0.06
1.29	0.96	0.73	0.56	0.44	0.35	0.28	0.23	0.19	0.16	0.13	0.11	0.09	0.08	0.07	0.06
1.32	0.98	0.75	0.58	0.45	0.36	0.29	0.24	0.19	0.16	0.14	0.11	0.10	0.08	0.07	0.06
1.36	1.01	0.76	0.59	0.46	0.37	0.30	0.24	0.20	0.17	0.14	0.12	0.10	0.08	0.07	0.06
0.8125	0.875	0.9375	1	1.0625	1.125	1.1875	1.25	1.3125	1.375	1.4375	1.5	1.5625	1.625	1.6875	1.75

Stress at end of Torsion Bar

L (in)	Stress in Torsion Bar (psi)											
0.25	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
0.5	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
0.75	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
1	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
1.25	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
1.5	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
1.75	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
2	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
2.25	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
2.5	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
2.75	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
3	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
3.25	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
3.5	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
3.75	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
4	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
4.25	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
4.5	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
4.75	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
5	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
5.25	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
5.5	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
5.75	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
6	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
6.25	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
6.5	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
6.75	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
7	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
7.25	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
7.5	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
7.75	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
8	594336888	27515597	4465341	1451018	641763.2	338152.5	199502.2	127387	86234.51	61060.34	44804.55	33842.98
D of torsion bar(in)	0.0625	0.125	0.1875	0.25	0.3125	0.375	0.4375	0.5	0.5625	0.625	0.6875	0.75
side dimension of mounting end (in):	0.0125	0.075	0.1375	0.2	0.2625	0.325	0.3875	0.45	0.5125	0.575	0.6375	0.7
d_{equiv} of mounting end (in):	0.01009905	0.060594	0.11109	0.161585	0.21208	0.262575	0.313071	0.363566	0.414061	0.464556	0.515052	0.565547
corrected endurance limit (psi)-->based on d_{equiv}	35680.2126	35680.21	35680.21	35680.21	35680.21	35680.21	34703.23	34203.51	33774.74	33399.85	33067.22	32768.59
uncorrected endurance limit:	92.5											
Correction Factors:	C_{load}	C_{surf}	C_{temp}	C_{relab} (99%)								
	0.7	0.67696	1	0.814								

$$S_T = \frac{16T}{\pi D^3} \quad (14)$$

26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
26184.43	20672.88	16605.72	13539.17	11183.5	9344.097	7886.94	6717.675	5768.575	4990.167	4345.74	3807.665	3354.877	2971.126	2643.74	2362.74
0.8125	0.875	0.9375	1	1.0625	1.125	1.1875	1.25	1.3125	1.375	1.4375	1.5	1.5625	1.625	1.6875	1.75
0.7625	0.825	0.8875	0.95	1.0125	1.075	1.1375	1.2	1.2625	1.325	1.3875	1.45	1.5125	1.575	1.6375	1.7
0.616042	0.666537	0.717033	0.767528	0.818023	0.868519	0.919014	0.969509	1.020004	1.0705	1.120995	1.17149	1.221985	1.272481	1.322976	1.373471
32497.88	32250.48	32022.85	31812.15	31616.15	31432.99	31261.15	31099.38	30946.59	30801.89	30664.48	30533.71	30408.98	30289.77	30175.65	30066.21

Safety Factor

L (in)	Safety Factor of Torsion Bar											
0.25	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
0.5	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
0.75	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
1	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
1.25	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
1.5	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
1.75	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
2	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
2.25	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
2.5	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
2.75	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
3	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
3.25	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
3.5	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
3.75	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
4	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
4.25	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
4.5	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
4.75	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
5	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
5.25	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
5.5	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
5.75	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
6	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
6.25	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
6.5	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
6.75	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
7	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
7.25	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
7.5	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
7.75	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
8	3.75E-05	1.09E-03	5.61E-03	1.62E-02	3.47E-02	6.26E-02	9.97E-02	1.49E-01	2.12E-01	2.89E-01	3.81E-01	4.91E-01
D of torsion bar(in)	0.0625	0.125	0.1875	0.25	0.3125	0.375	0.4375	0.5	0.5625	0.625	0.6875	0.75
side dimension of mounting end (in):	0.0125	0.075	0.1375	0.2	0.2625	0.325	0.3875	0.45	0.5125	0.575	0.6375	0.7
d_{equiv} of mounting end (in):	0.010099053	0.060594	0.11109	0.161585	0.21208	0.262575	0.313071	0.363566	0.414061	0.464556	0.515052	0.565547
D/d	6.188699379	2.0629	1.687827	1.547175	1.4735	1.428161	1.397448	1.375267	1.358495	1.345369	1.334818	1.32615
A value	-358.989117	0.717842	1.060509	0.984437	0.932836	0.902	0.882741	0.870055	0.861306	0.855036	0.8504	0.846879
b value	-307.185074	-0.35052	-0.08185	-0.14123	-0.17881	-0.19988	-0.21219	-0.21974	-0.22457	-0.22776	-0.22991	-0.23139
notch radius (in)	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
Geometric stress concentration factor (K_t):	-4.254E-119	0.979044	1.198208	1.281306	1.367237	1.443256	1.509213	1.56685	1.617847	1.663536	1.704935	1.742813
Notch sensitivity (q)	0.86821433	0.868214	0.868214	0.868214	0.868214	0.868214	0.868214	0.868214	0.868214	0.868214	0.868214	0.868214
fatigue stress-concentration factor (K_{fs})	0.13178567	0.981806	1.172087	1.244234	1.31884	1.384841	1.442106	1.492147	1.536423	1.576092	1.612035	1.644921
corrected endurance limit (psi)-->based on d_{equiv}	35680.21263	35680.21	35680.21	35680.21	35680.21	35680.21	34703.23	34203.51	33774.74	33399.85	33067.22	32768.59

6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.569622	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
6.19E-01	7.66E-01	9.34E-01	1.12E+00	1.33E+00	1.57E+00	1.83E+00	2.12E+00	2.43E+00	2.77E+00	3.14E+00	3.54E+00	3.96E+00	4.42E+00	4.92E+00	5.44E+00
0.8125	0.875	0.9375	1	1.0625	1.125	1.1875	1.25	1.3125	1.375	1.4375	1.5	1.5625	1.625	1.6875	1.75
0.7625	0.825	0.8875	0.95	1.0125	1.075	1.1375	1.2	1.2625	1.325	1.3875	1.45	1.5125	1.575	1.6375	1.7
0.616042	0.666537	0.717033	0.767528	0.818023	0.868519	0.919014	0.969509	1.020004	1.0705	1.120995	1.17149	1.221985	1.272481	1.322976	1.373471
1.318903	1.312754	1.307472	1.302884	1.298863	1.295309	1.292146	1.289312	1.286759	1.284447	1.282343	1.280421	1.278657	1.277033	1.275533	1.274144
0.844145	0.841982	0.840243	0.838824	0.837651	0.836672	0.835847	0.835144	0.834541	0.834021	0.833568	0.833173	0.832824	0.832517	0.832244	0.832
-0.23242	-0.23314	-0.23364	-0.23399	-0.23422	-0.23437	-0.23446	-0.2345	-0.23451	-0.23449	-0.23446	-0.23441	-0.23435	-0.23428	-0.23421	-0.23413
0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
1.777763	1.810241	1.840609	1.869155	1.896112	1.921671	1.945991	1.969202	1.991417	2.012732	2.033228	2.052976	2.072037	2.090466	2.108311	2.125612
0.868214	0.868214	0.868214	0.868214	0.868214	0.868214	0.868214	0.868214	0.868214	0.868214	0.868214	0.868214	0.868214	0.868214	0.868214	0.868214
1.675265	1.703463	1.729829	1.754613	1.778017	1.800208	1.821323	1.841475	1.860763	1.879269	1.897063	1.914209	1.930758	1.946758	1.962251	1.977273
32497.88	32250.48	32022.85	31812.15	31616.15	31432.99	31261.15	31099.38	30946.59	30801.89	30664.48	30533.71	30408.98	30289.77	30175.65	30066.21

6063 Aluminum

Angle of Deflection

L (in)	angle of twist in degrees													
2	22647.02	1415.44	279.59	88.46	36.24	17.47	9.43	5.53	3.45	2.26	1.55	1.09	0.79	0.59
3	33970.53	2123.16	419.39	132.70	54.35	26.21	14.15	8.29	5.18	3.40	2.32	1.64	1.19	0.88
3.25	36801.41	2300.09	454.34	143.76	58.88	28.40	15.33	8.98	5.61	3.68	2.51	1.77	1.29	0.96
3.5	39632.29	2477.02	489.29	154.81	63.41	30.58	16.51	9.68	6.04	3.96	2.71	1.91	1.39	1.03
3.75	42463.17	2653.95	524.24	165.87	67.94	32.76	17.69	10.37	6.47	4.25	2.90	2.05	1.49	1.11
4	45294.04	2830.88	559.19	176.93	72.47	34.95	18.86	11.06	6.90	4.53	3.09	2.18	1.59	1.18
4.25	48124.92	3007.81	594.13	187.99	77.00	37.13	20.04	11.75	7.33	4.81	3.29	2.32	1.68	1.25
4.5	50955.80	3184.74	629.08	199.05	81.53	39.32	21.22	12.44	7.77	5.10	3.48	2.46	1.78	1.33
4.75	53786.68	3361.67	664.03	210.10	86.06	41.50	22.40	13.13	8.20	5.38	3.67	2.59	1.88	1.40
5	56617.55	3538.60	698.98	221.16	90.59	43.69	23.58	13.82	8.63	5.66	3.87	2.73	1.98	1.47
5.25	59448.43	3715.53	733.93	232.22	95.12	45.87	24.76	14.51	9.06	5.94	4.06	2.87	2.08	1.55
5.5	62279.31	3892.46	768.88	243.28	99.65	48.06	25.94	15.20	9.49	6.23	4.25	3.00	2.18	1.62
5.75	65110.19	4069.39	803.83	254.34	104.18	50.24	27.12	15.90	9.92	6.51	4.45	3.14	2.28	1.69
6	67941.07	4246.32	838.78	265.39	108.71	52.42	28.30	16.59	10.36	6.79	4.64	3.28	2.38	1.77
6.25	70771.94	4423.25	873.73	276.45	113.24	54.61	29.48	17.28	10.79	7.08	4.83	3.41	2.48	1.84
6.5	73602.82	4600.18	908.68	287.51	117.76	56.79	30.66	17.97	11.22	7.36	5.03	3.55	2.58	1.92
6.75	76433.70	4777.11	943.63	298.57	122.29	58.98	31.83	18.66	11.65	7.64	5.22	3.69	2.68	1.99
7	79264.58	4954.04	978.58	309.63	126.82	61.16	33.01	19.35	12.08	7.93	5.41	3.82	2.78	2.06
7.25	82095.45	5130.97	1013.52	320.69	131.35	63.35	34.19	20.04	12.51	8.21	5.61	3.96	2.87	2.14
7.5	84926.33	5307.90	1048.47	331.74	135.88	65.53	35.37	20.73	12.94	8.49	5.80	4.10	2.97	2.21
7.75	87757.21	5484.83	1083.42	342.80	140.41	67.71	36.55	21.43	13.38	8.78	5.99	4.23	3.07	2.28
8	90588.09	5661.76	1118.37	353.86	144.94	69.90	37.73	22.12	13.81	9.06	6.19	4.37	3.17	2.36
8.25	93418.96	5838.69	1153.32	364.92	149.47	72.08	38.91	22.81	14.24	9.34	6.38	4.51	3.27	2.43
8.5	96249.84	6015.62	1188.27	375.98	154.00	74.27	40.09	23.50	14.67	9.62	6.57	4.64	3.37	2.51
8.75	99080.72	6192.55	1223.22	387.03	158.53	76.45	41.27	24.19	15.10	9.91	6.77	4.78	3.47	2.58
9	101911.60	6369.47	1258.17	398.09	163.06	78.64	42.45	24.88	15.53	10.19	6.96	4.91	3.57	2.65
9.25	104742.48	6546.40	1293.12	409.15	167.59	80.82	43.62	25.57	15.96	10.47	7.15	5.05	3.67	2.73
9.5	107573.35	6723.33	1328.07	420.21	172.12	83.00	44.80	26.26	16.40	10.76	7.35	5.19	3.77	2.80
9.75	110404.23	6900.26	1363.02	431.27	176.65	85.19	45.98	26.95	16.83	11.04	7.54	5.32	3.87	2.87
10	113235.11	7077.19	1397.96	442.32	181.18	87.37	47.16	27.65	17.26	11.32	7.73	5.46	3.96	2.95
10.25	116065.99	7254.12	1432.91	453.38	185.71	89.56	48.34	28.34	17.69	11.61	7.93	5.60	4.06	3.02
10.5	118896.86	7431.05	1467.86	464.44	190.23	91.74	49.52	29.03	18.12	11.89	8.12	5.73	4.16	3.09
D (in)	0.0625	0.1250	0.1875	0.2500	0.3125	0.3750	0.4375	0.5000	0.5625	0.6250	0.6875	0.7500	0.8125	0.8750

0.45	0.35	0.27	0.22	0.17	0.14	0.12	0.10	0.08	0.07	0.06	0.05	0.04	0.04
0.67	0.52	0.41	0.32	0.26	0.21	0.17	0.15	0.12	0.10	0.09	0.07	0.06	0.06
0.73	0.56	0.44	0.35	0.28	0.23	0.19	0.16	0.13	0.11	0.09	0.08	0.07	0.06
0.78	0.60	0.47	0.38	0.30	0.25	0.20	0.17	0.14	0.12	0.10	0.09	0.07	0.06
0.84	0.65	0.51	0.40	0.33	0.27	0.22	0.18	0.15	0.13	0.11	0.09	0.08	0.07
0.89	0.69	0.54	0.43	0.35	0.28	0.23	0.19	0.16	0.14	0.12	0.10	0.09	0.07
0.95	0.73	0.58	0.46	0.37	0.30	0.25	0.21	0.17	0.15	0.12	0.11	0.09	0.08
1.01	0.78	0.61	0.49	0.39	0.32	0.26	0.22	0.18	0.15	0.13	0.11	0.10	0.08
1.06	0.82	0.64	0.51	0.41	0.34	0.28	0.23	0.19	0.16	0.14	0.12	0.10	0.09
1.12	0.86	0.68	0.54	0.43	0.35	0.29	0.24	0.20	0.17	0.14	0.12	0.11	0.09
1.17	0.91	0.71	0.57	0.46	0.37	0.31	0.25	0.21	0.18	0.15	0.13	0.11	0.10
1.23	0.95	0.75	0.59	0.48	0.39	0.32	0.27	0.22	0.19	0.16	0.14	0.12	0.10
1.29	0.99	0.78	0.62	0.50	0.41	0.33	0.28	0.23	0.20	0.17	0.14	0.12	0.11
1.34	1.04	0.81	0.65	0.52	0.42	0.35	0.29	0.24	0.20	0.17	0.15	0.13	0.11
1.40	1.08	0.85	0.67	0.54	0.44	0.36	0.30	0.25	0.21	0.18	0.15	0.13	0.12
1.45	1.12	0.88	0.70	0.56	0.46	0.38	0.31	0.26	0.22	0.19	0.16	0.14	0.12
1.51	1.17	0.92	0.73	0.59	0.48	0.39	0.33	0.27	0.23	0.20	0.17	0.14	0.12
1.57	1.21	0.95	0.76	0.61	0.50	0.41	0.34	0.28	0.24	0.20	0.17	0.15	0.13
1.62	1.25	0.98	0.78	0.63	0.51	0.42	0.35	0.29	0.25	0.21	0.18	0.15	0.13
1.68	1.30	1.02	0.81	0.65	0.53	0.44	0.36	0.30	0.26	0.22	0.19	0.16	0.14
1.73	1.34	1.05	0.84	0.67	0.55	0.45	0.37	0.31	0.26	0.22	0.19	0.17	0.14
1.79	1.38	1.08	0.86	0.70	0.57	0.47	0.39	0.32	0.27	0.23	0.20	0.17	0.15
1.85	1.43	1.12	0.89	0.72	0.58	0.48	0.40	0.33	0.28	0.24	0.20	0.18	0.15
1.90	1.47	1.15	0.92	0.74	0.60	0.49	0.41	0.34	0.29	0.25	0.21	0.18	0.16
1.96	1.51	1.19	0.94	0.76	0.62	0.51	0.42	0.35	0.30	0.25	0.22	0.19	0.16
2.01	1.56	1.22	0.97	0.78	0.64	0.52	0.44	0.36	0.31	0.26	0.22	0.19	0.17
2.07	1.60	1.25	1.00	0.80	0.65	0.54	0.45	0.37	0.32	0.27	0.23	0.20	0.17
2.12	1.64	1.29	1.02	0.83	0.67	0.55	0.46	0.38	0.32	0.28	0.24	0.20	0.18
2.18	1.68	1.32	1.05	0.85	0.69	0.57	0.47	0.39	0.33	0.28	0.24	0.21	0.18
2.24	1.73	1.36	1.08	0.87	0.71	0.58	0.48	0.40	0.34	0.29	0.25	0.21	0.18
2.29	1.77	1.39	1.11	0.89	0.73	0.60	0.50	0.41	0.35	0.30	0.25	0.22	0.19
2.35	1.81	1.42	1.13	0.91	0.74	0.61	0.51	0.42	0.36	0.30	0.26	0.22	0.19
0.9375	1.0000	1.0625	1.1250	1.1875	1.2500	1.3125	1.3750	1.4375	1.5000	1.5625	1.6250	1.6875	1.7500

Stress at end of Torsion Bar

L (in)	Stress in Torsion Bar (psi)													
0.25	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
0.5	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
0.75	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
1	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
1.25	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
1.5	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
1.75	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
2	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
2.25	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
2.5	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
2.75	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
3	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
3.25	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
3.5	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
3.75	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
4	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
4.25	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
4.5	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
4.75	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
5	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
5.25	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
5.5	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
5.75	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
6	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
6.25	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
6.5	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
6.75	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
7	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
7.25	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
7.5	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
7.75	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
8	7.83E+08	27014972	5E+06	2E+06	846383	468287	287703	190080	132493	96237	72226	55669	43866	35215
D of torsion bar(in)	0.0625	0.1250	0.1875	0.2500	0.3125	0.3750	0.4375	0.5000	0.5625	0.6250	0.6875	0.7500	0.8125	0.8750
side dimension of mounting end (in):	0.0125	0.0750	0.1375	0.2000	0.2625	0.3250	0.3875	0.4500	0.5125	0.5750	0.6375	0.7000	0.7625	0.8250
dequiv of mounting end (in):	0.010099	0.0605943	0.1111	0.1616	0.2121	0.2626	0.3131	0.3636	0.4141	0.4646	0.5151	0.5655	0.616	0.6665
D/d	6.188699	2.0628998	1.6878	1.5472	1.4735	1.4282	1.3974	1.3753	1.3585	1.3454	1.3348	1.3261	1.3189	1.3128
A value	-358.9891	0.7178423	1.0605	0.9844	0.9328	0.902	0.8827	0.8701	0.8613	0.855	0.8504	0.8469	0.8441	0.842
b value	-307.1851	-0.350523	-0.0819	-0.1412	-0.1788	-0.1999	-0.2122	-0.2197	-0.2246	-0.2278	-0.2299	-0.2314	-0.2324	-0.2331
notch radius (in)	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
Geometric stress concentration factor (Kt):	-4.3E-119	0.9790441	1.1982	1.2813	1.3672	1.4433	1.5092	1.5669	1.6178	1.6635	1.7049	1.7428	1.7778	1.8102
Notch sensitivity (q)	0.868214	0.8682143	0.8682	0.8682	0.8682	0.8682	0.8682	0.8682	0.8682	0.8682	0.8682	0.8682	0.8682	0.8682
fatigue stress-concentration factor (Kfs)	0.131786	0.9818058	1.1721	1.2442	1.3188	1.3848	1.4421	1.4921	1.5364	1.5761	1.612	1.6449	1.6753	1.7035
corrected endurance limit (psi)-->based on d	19267.59	19267.588	19268	19268	19268	19268	18740	18470	18239	18036	17857	17695	17549	17416

28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
28725	23756	19884	16821	14365	12370	10734	9377.9	8244.1	7288.7	6477.5	5784.1	5187.7	4671.8	4223.1	3831
0.9375	1.0000	1.0625	1.1250	1.1875	1.2500	1.3125	1.3750	1.4375	1.5000	1.5625	1.6250	1.6875	1.7500	1.8125	1.8750
0.8875	0.9500	1.0125	1.0750	1.1375	1.2000	1.2625	1.3250	1.3875	1.4500	1.5125	1.5750	1.6375	1.7000	1.7625	1.8250
0.717	0.7675	0.818	0.8685	0.919	0.9695	1.02	1.0705	1.121	1.1715	1.222	1.2725	1.323	1.3735	1.424	1.4745
1.3075	1.3029	1.2989	1.2953	1.2921	1.2893	1.2868	1.2844	1.2823	1.2804	1.2787	1.277	1.2755	1.2741	1.2729	1.2717
0.8402	0.8388	0.8377	0.8367	0.8358	0.8351	0.8345	0.834	0.8336	0.8332	0.8328	0.8325	0.8322	0.832	0.8318	0.8316
-0.2336	-0.234	-0.2342	-0.2344	-0.2345	-0.2345	-0.2345	-0.2345	-0.2345	-0.2344	-0.2343	-0.2343	-0.2342	-0.2341	-0.2341	-0.234
0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
1.8406	1.8692	1.8961	1.9217	1.946	1.9692	1.9914	2.0127	2.0332	2.053	2.072	2.0905	2.1083	2.1256	2.1424	2.1587
0.8682	0.8682	0.8682	0.8682	0.8682	0.8682	0.8682	0.8682	0.8682	0.8682	0.8682	0.8682	0.8682	0.8682	0.8682	0.8682
1.7298	1.7546	1.778	1.8002	1.8213	1.8415	1.8608	1.8793	1.8971	1.9142	1.9308	1.9468	1.9623	1.9773	1.9919	2.006
17293	17179	17073	16974	16881	16794	16711	16633	16559	16488	16421	16357	16295	16236	16179	16125

Safety Factor

L (in)	Safety Factor of Torsion Bar												
0.25	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
0.5	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
0.75	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
1	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
1.25	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
1.5	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
1.75	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
2	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
2.25	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
2.5	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
2.75	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
3	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
3.25	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
3.5	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
3.75	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
4	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
4.25	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
4.5	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
4.75	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
5	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
5.25	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
5.5	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
5.75	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
6	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
6.25	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
6.5	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
6.75	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
7	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
7.25	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
7.5	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
7.75	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
8	1.93E-05	5.59E-04	2.89E-03	8.37E-03	1.79E-02	3.23E-02	5.14E-02	7.69E-02	1.09E-01	1.49E-01	1.97E-01	2.54E-01	3.20E-01
D of torsion bar(in)	0.0625	0.1250	0.1875	0.2500	0.3125	0.3750	0.4375	0.5000	0.5625	0.6250	0.6875	0.7500	0.8125
side dimension of mounting end (in):	0.0125	0.0750	0.1375	0.2000	0.2625	0.3250	0.3875	0.4500	0.5125	0.5750	0.6375	0.7000	0.7625
dequiv of mounting end (in):	0.0100991	0.060594315	0.11109	0.16158	0.21208	0.26258	0.31307	0.36357	0.41406	0.46455642	0.51505	0.56555	0.61604
D/d	6.1886994	2.062899793	1.68783	1.54717	1.4735	1.42816	1.39745	1.37527	1.35849	1.34536943	1.33482	1.32615	1.3189
A value	-358.9891	0.717842293	1.06051	0.98444	0.93284	0.902	0.88274	0.87006	0.86131	0.85503633	0.8504	0.84688	0.84415
b value	-307.1851	-0.3505229	-0.0819	-0.1412	-0.1788	-0.1999	-0.2122	-0.2197	-0.2246	-0.2277583	-0.2299	-0.2314	-0.2324
notch radius (in)	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
Geometric stress concentration factor (Kt):	-4.3E-119	0.979044087	1.19821	1.28131	1.36724	1.44326	1.50921	1.56685	1.61785	1.6635363	1.70493	1.74281	1.77776
Notch sensitivity (q)	0.8682143	0.86821433	0.86821	0.86821	0.86821	0.86821	0.86821	0.86821	0.86821	0.86821433	0.86821	0.86821	0.86821
fatigue stress-concentration factor (Kfs)	0.1317857	0.981805776	1.17209	1.24423	1.31884	1.38484	1.44211	1.49215	1.53642	1.57609172	1.61203	1.64492	1.67526
corrected endurance limit (psi)-->based on d	19267.588	19267.58812	19267.6	19267.6	19267.6	19267.6	18740	18470.2	18238.6	18036.1733	17856.6	17695.3	17549.1

3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
3.96E-01	4.83E-01	5.81E-01	6.90E-01	8.12E-01	9.47E-01	1.09E+00	1.26E+00	1.43E+00	1.62E+00	1.83E+00	2.05E+00	2.29E+00	2.55E+00	2.82E+00	3.11E+00	3.42E+00
0.8750	0.9375	1.0000	1.0625	1.1250	1.1875	1.2500	1.3125	1.3750	1.4375	1.5000	1.5625	1.6250	1.6875	1.7500	1.8125	1.8750
0.8250	0.8875	0.9500	1.0125	1.0750	1.1375	1.2000	1.2625	1.3250	1.3875	1.4500	1.5125	1.5750	1.6375	1.7000	1.7625	1.8250
0.66654	0.71703	0.767528	0.818023	0.86852	0.91901	0.96951	1.02	1.0705	1.12099	1.17149	1.22199	1.27248	1.32298	1.37347	1.42397	1.47446
1.31275	1.30747	1.302884	1.298863	1.29531	1.29215	1.28931	1.28676	1.28445	1.28234	1.28042	1.27866	1.27703	1.27553	1.27414	1.27285	1.27165
0.84198	0.84024	0.838824	0.837651	0.83667	0.83585	0.83514	0.83454	0.83402	0.83357	0.83317	0.83282	0.83252	0.83224	0.832	0.83178	0.83159
-0.2331	-0.2336	-0.23399	-0.23422	-0.2344	-0.2345	-0.2345	-0.23451	-0.23449	-0.23446	-0.23441	-0.23435	-0.23428	-0.23421	-0.23413	-0.23405	-0.23397
0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
1.81024	1.84061	1.869155	1.896112	1.92167	1.94599	1.9692	1.99142	2.01273	2.03323	2.05298	2.07204	2.09047	2.10831	2.12561	2.14241	2.15874
0.86821	0.86821	0.868214	0.868214	0.86821	0.86821	0.86821	0.86821	0.86821	0.86821	0.86821	0.86821	0.86821	0.86821	0.86821	0.86821	0.86821
1.70346	1.72983	1.754613	1.778017	1.80021	1.82132	1.84148	1.86076	1.87927	1.89706	1.91421	1.93076	1.94676	1.96225	1.97727	1.99186	2.00603
17415.5	17292.6	17178.81	17072.96	16974.1	16881.3	16793.9	16711.4	16633.3	16559.1	16488.4	16421.1	16356.7	16295.1	16236	16179.2	16124.6